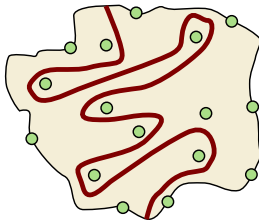


Topography and topology

Ian Short



<http://maths.org/~ims25/math/presentations/Bletchley2010.pdf>

INTRODUCTION

What is topography?

INTRODUCTION

●○○○○○○○○○○

WINDING NUMBERS

○○○○○○○○○○

DIVIDING WALLS

○○○○○○○○○○

EXOTIC ISLANDS

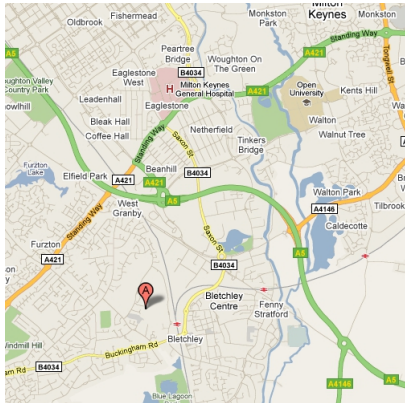
○○○○○○○○○○

TOPOGRAPHY

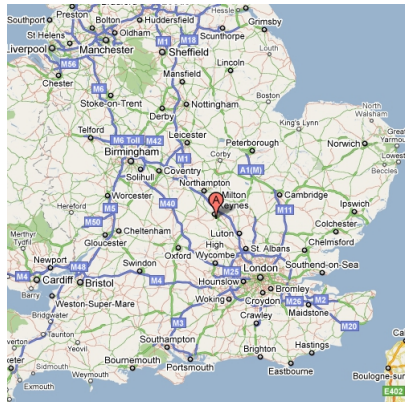
TOPOGRAPHY



SENSE OF DIRECTION



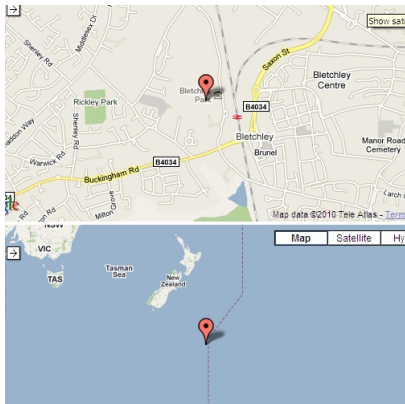
SENSE OF DIRECTION



SENSE OF DIRECTION

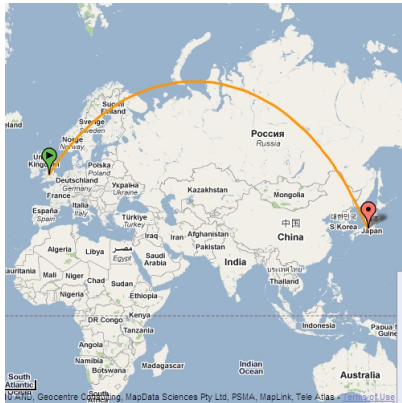


ANTIPODAL POINTS



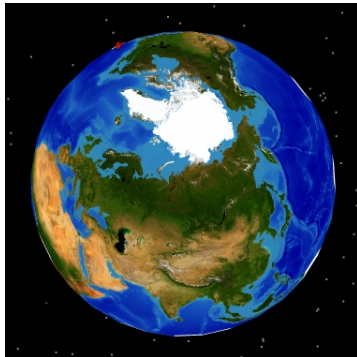
<http://www.antipodemap.com/>

GREAT CIRCLES



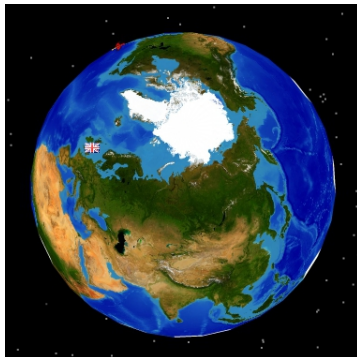
http://maps.forum.nu/gm_flight_path.html

GREAT CIRCLES



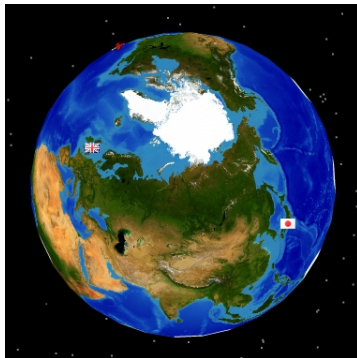
<http://www.earthbrowser.com/>

GREAT CIRCLES



<http://www.earthbrowser.com/>

GREAT CIRCLES



<http://www.earthbrowser.com/>

GREAT CIRCLES



<http://www.earthbrowser.com/>

GREAT CIRCLES



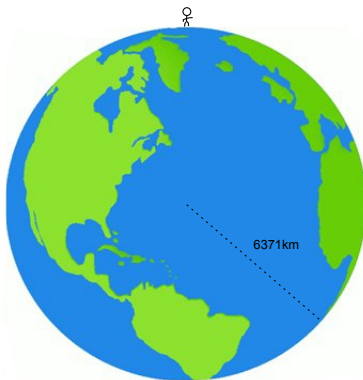
<http://www.earthbrowser.com/>

GREAT CIRCLES



<http://www.earthbrowser.com/>

HOW FAR TO THE HORIZON?



What is topology?

TOPOLOGY

No straight lines...

TOPOLOGY

No straight lines...

No angles...

TOPOLOGY

No straight lines...

No angles...

No distances...

TOPOLOGY

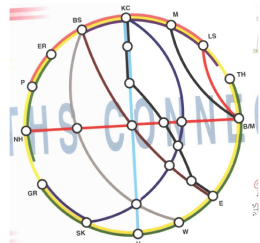
No straight lines...

No angles...

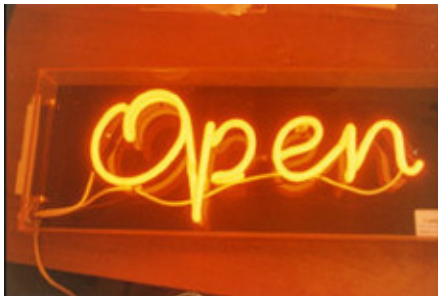
No distances...

Just *shapes* and *distortion* of shapes.

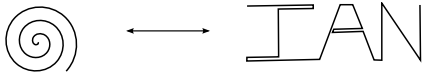
FROM *Get knotted!*



NEON SIGNS



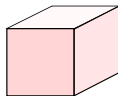
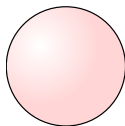
TOPOLOGY



TOPOLOGY



TOPOLOGY

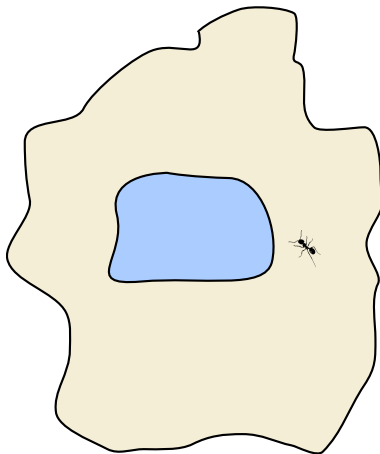


WINDING NUMBERS

THE JORDAN CUVE THEOREM

[YOUTUBE LINK](#)

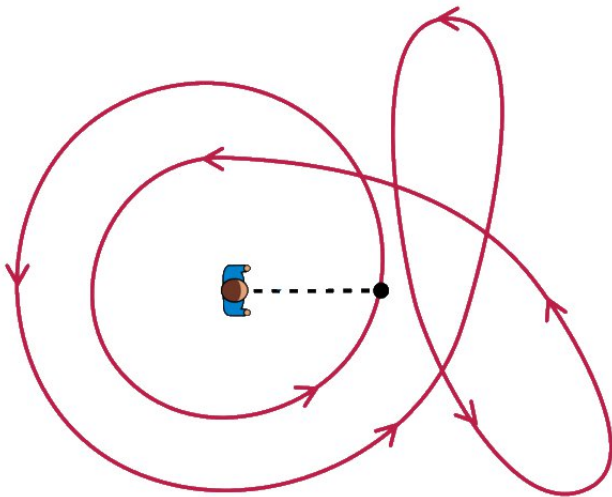
LAKE OR SEA?



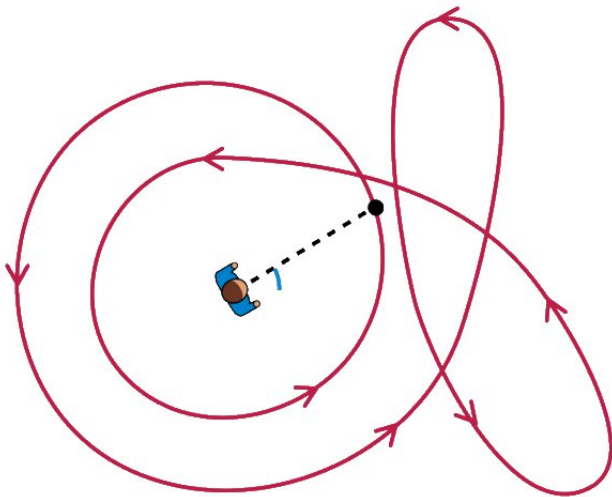
HANGING PROBLEM



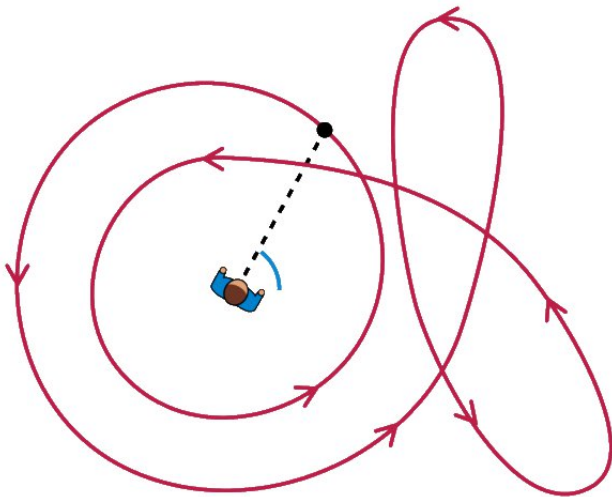
WINDING NUMBERS



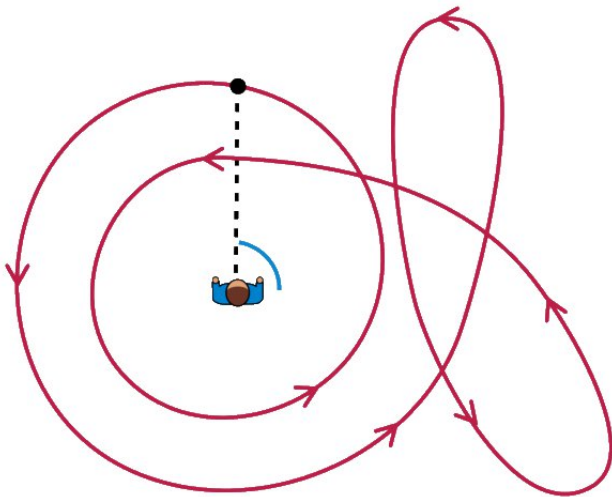
WINDING NUMBERS



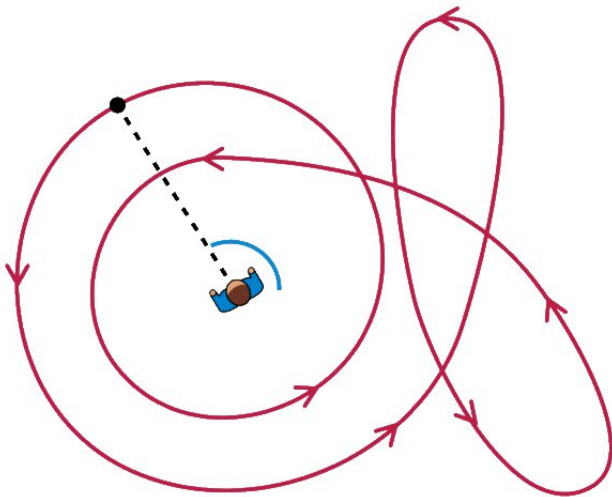
WINDING NUMBERS



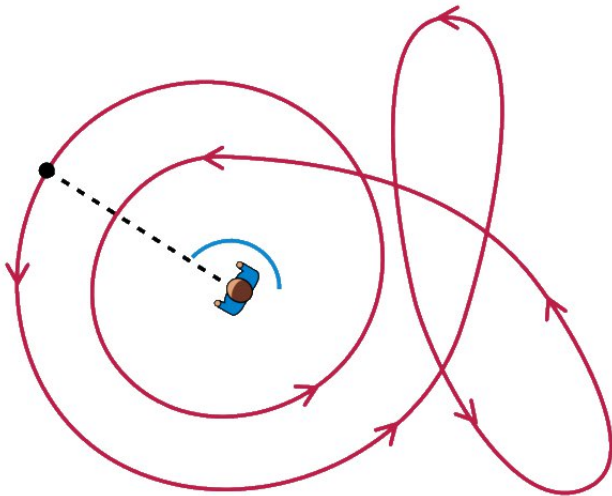
WINDING NUMBERS



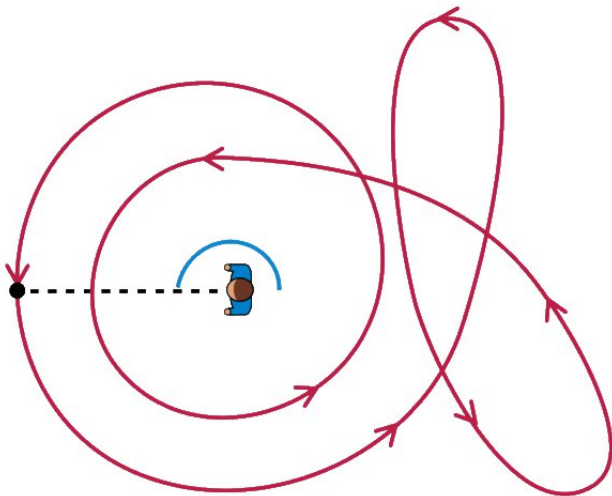
WINDING NUMBERS



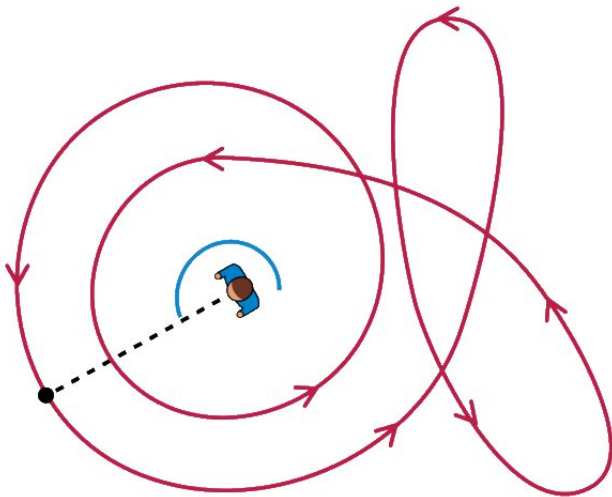
WINDING NUMBERS



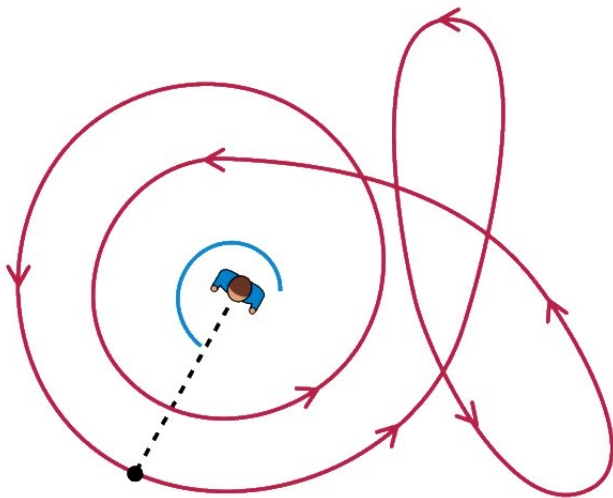
WINDING NUMBERS



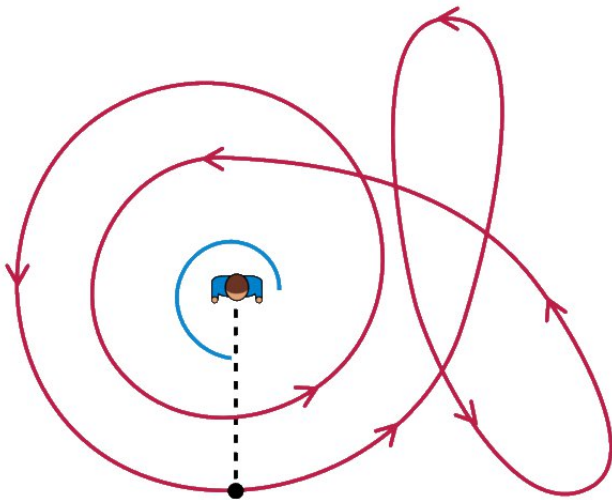
WINDING NUMBERS



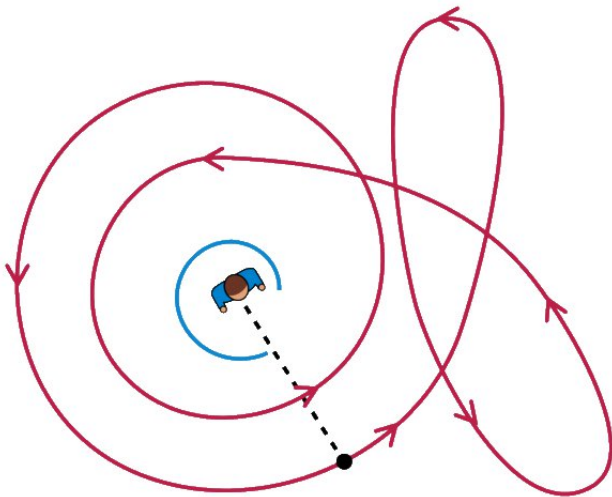
WINDING NUMBERS



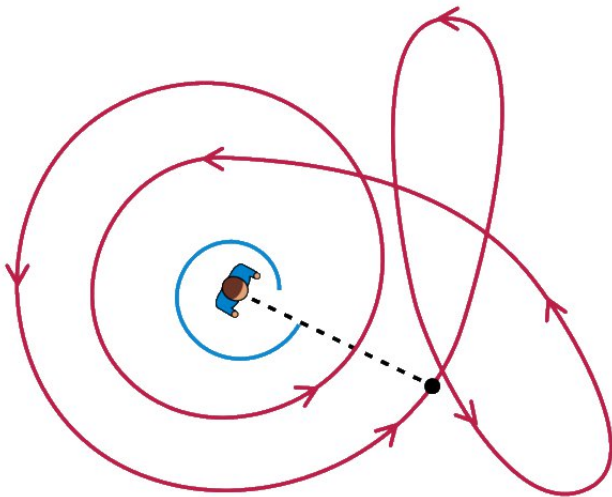
WINDING NUMBERS



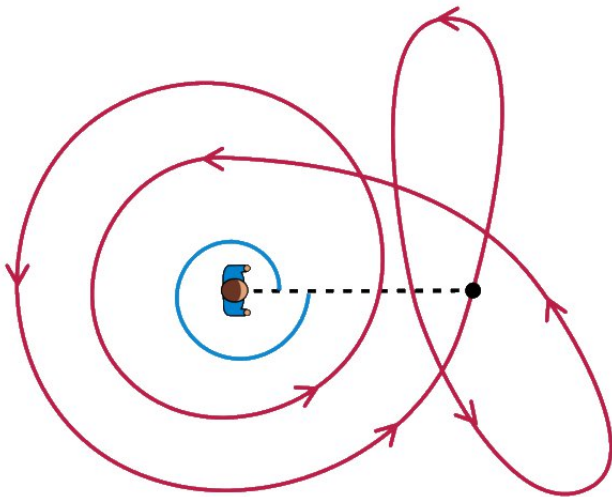
WINDING NUMBERS



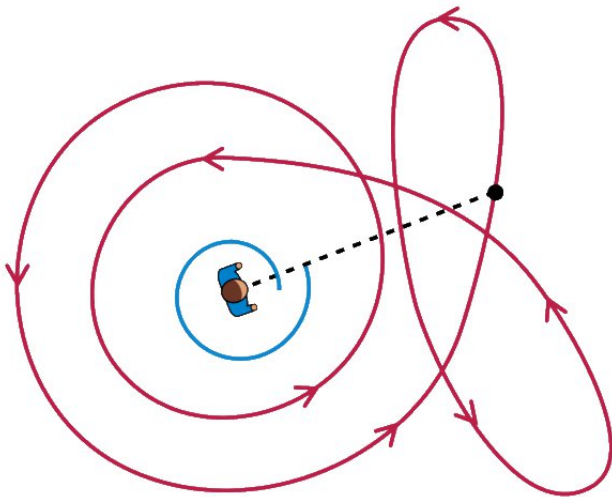
WINDING NUMBERS



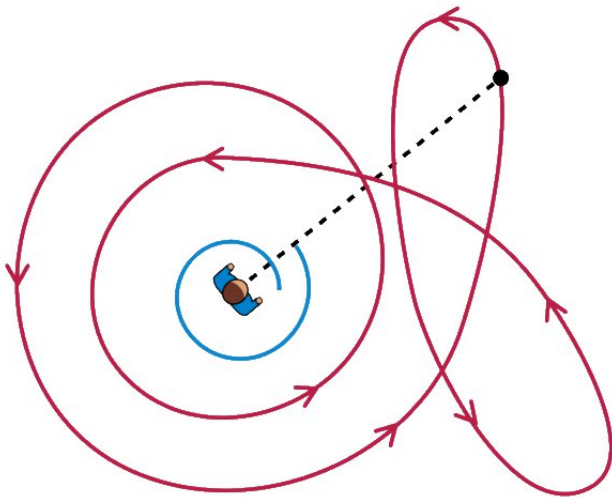
WINDING NUMBERS



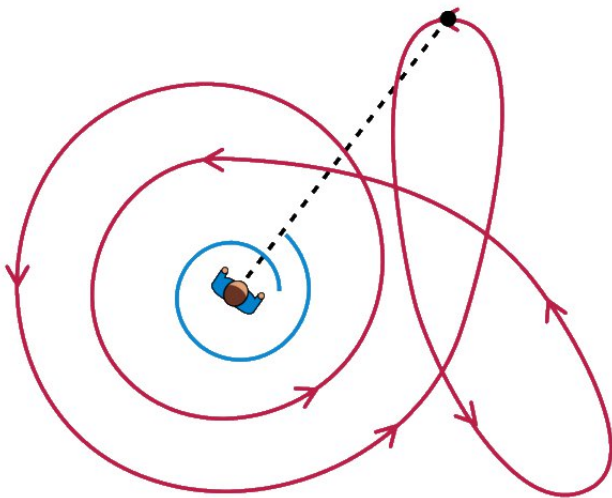
WINDING NUMBERS



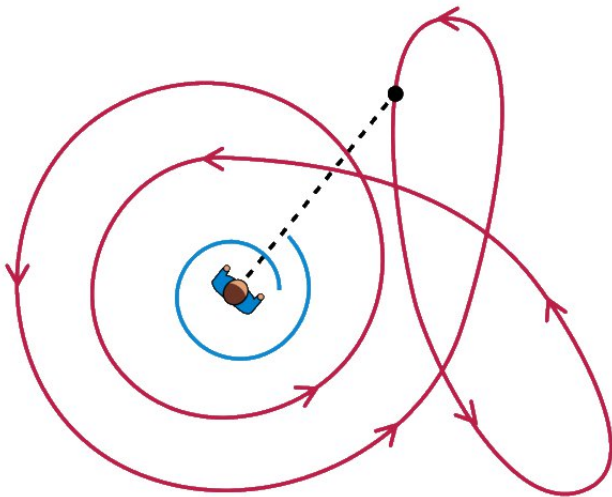
WINDING NUMBERS



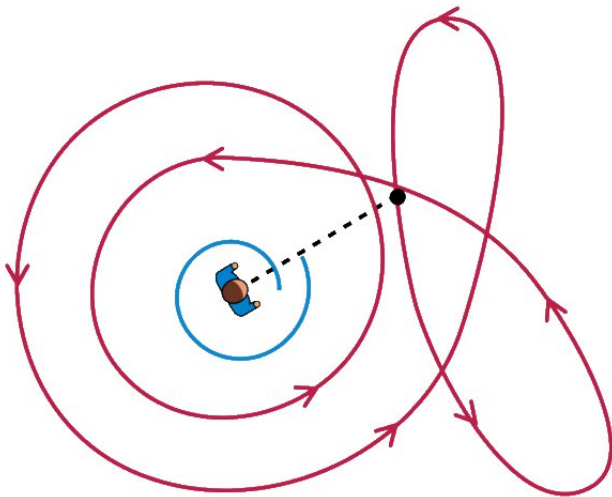
WINDING NUMBERS



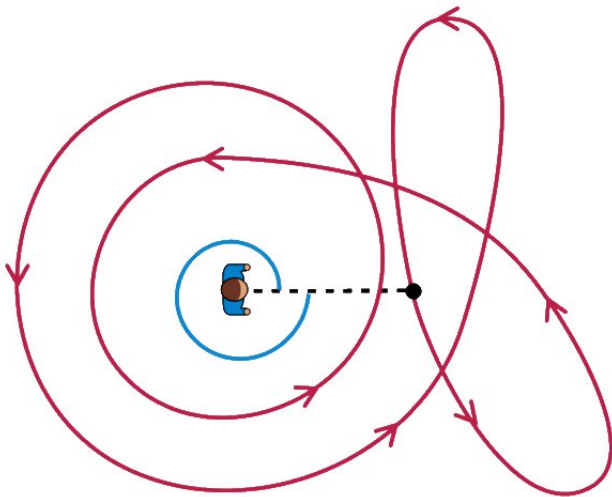
WINDING NUMBERS



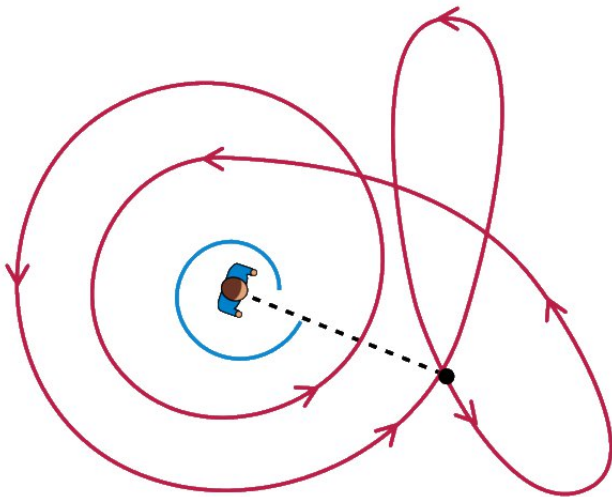
WINDING NUMBERS



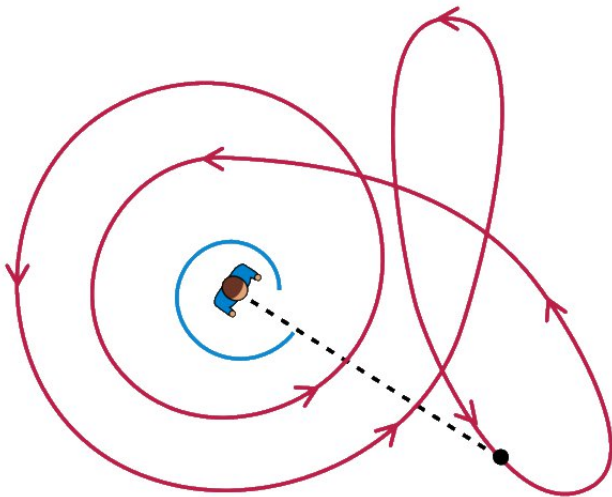
WINDING NUMBERS



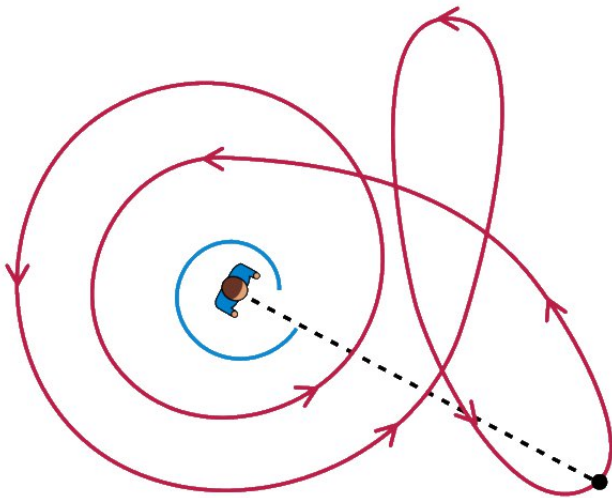
WINDING NUMBERS



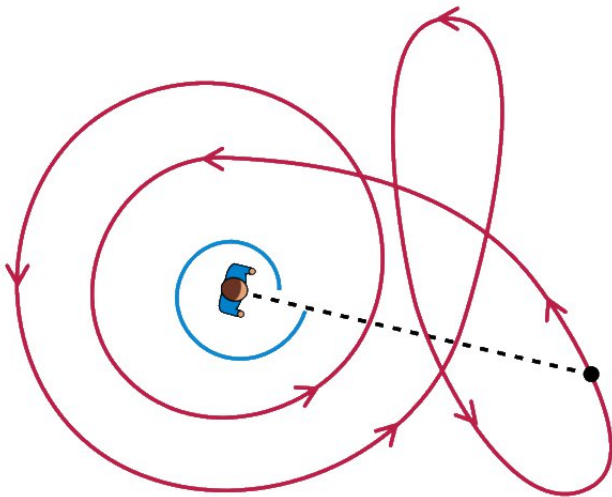
WINDING NUMBERS



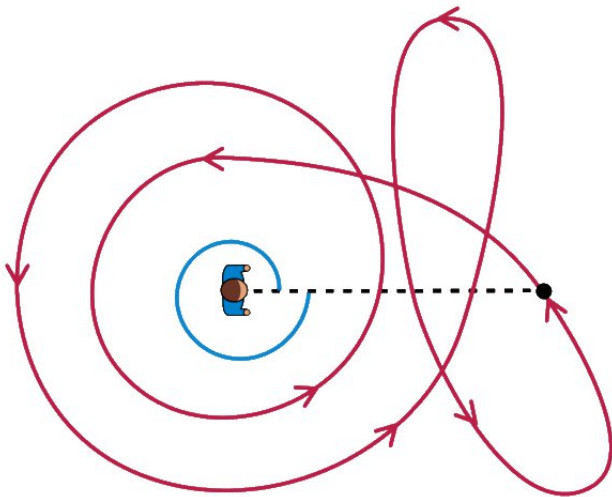
WINDING NUMBERS



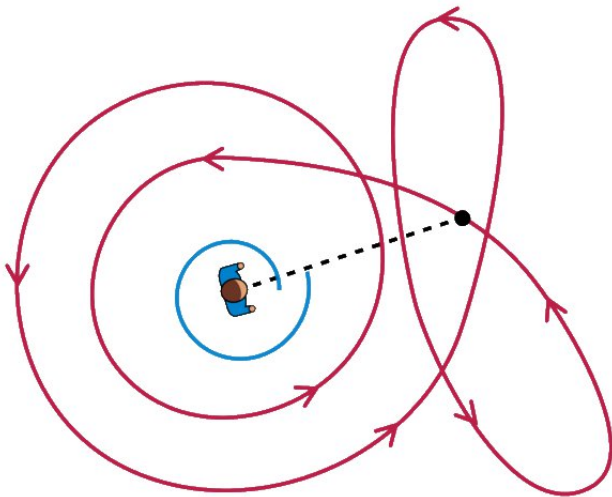
WINDING NUMBERS



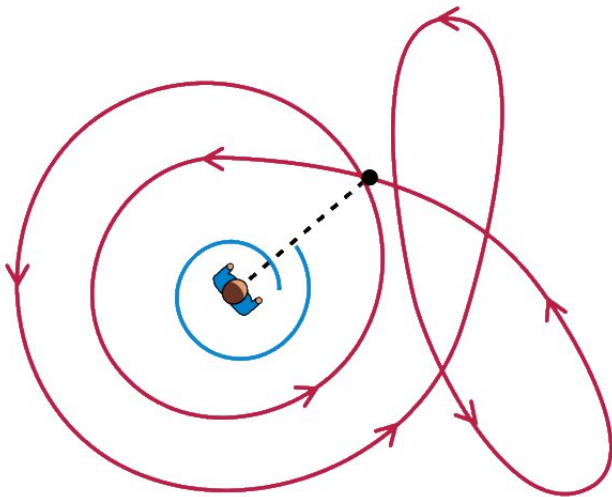
WINDING NUMBERS



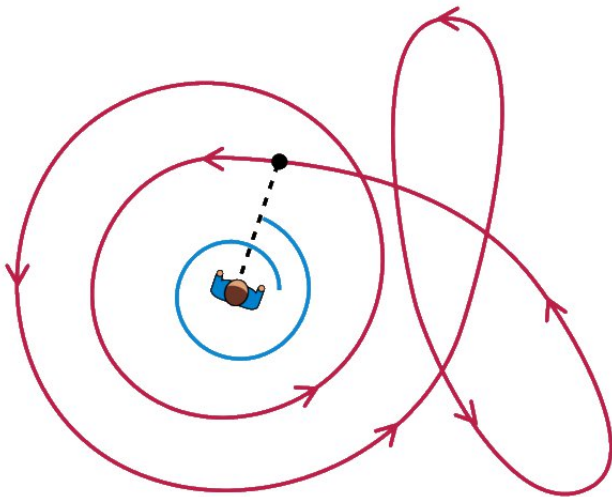
WINDING NUMBERS



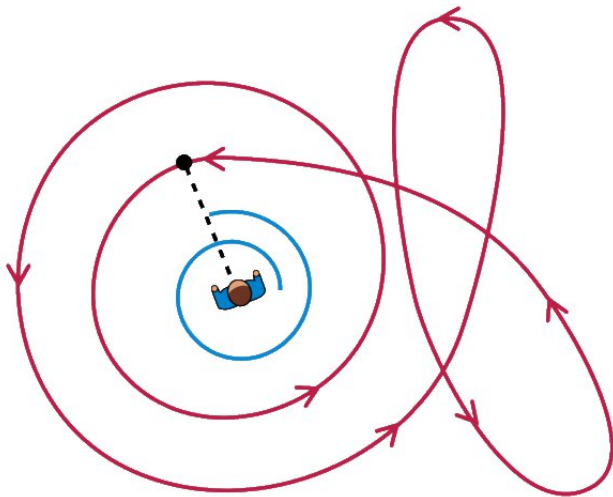
WINDING NUMBERS



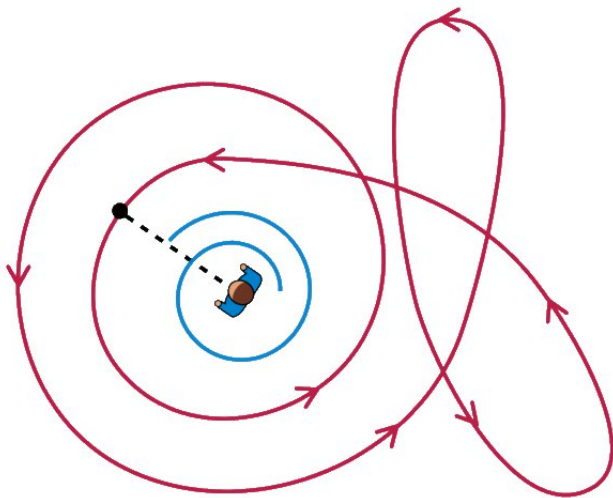
WINDING NUMBERS



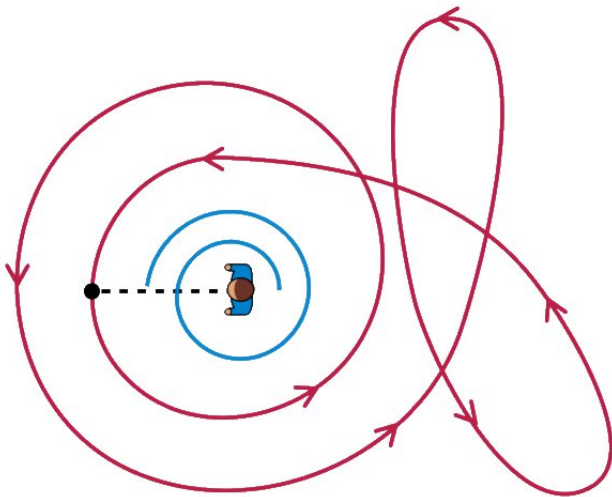
WINDING NUMBERS



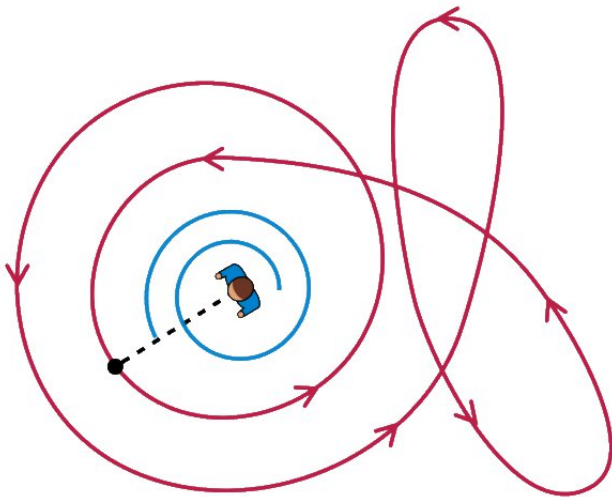
WINDING NUMBERS



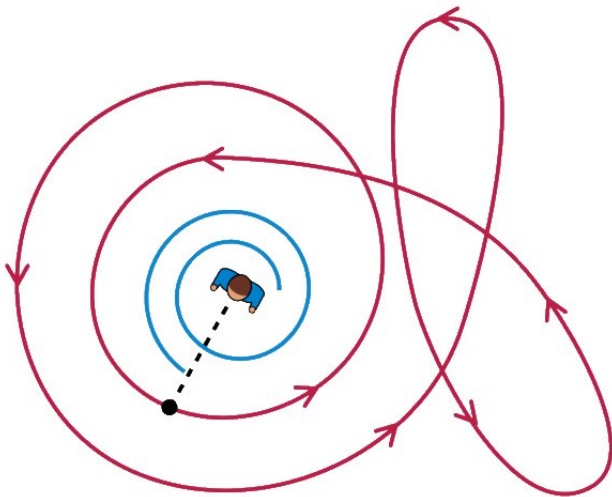
WINDING NUMBERS



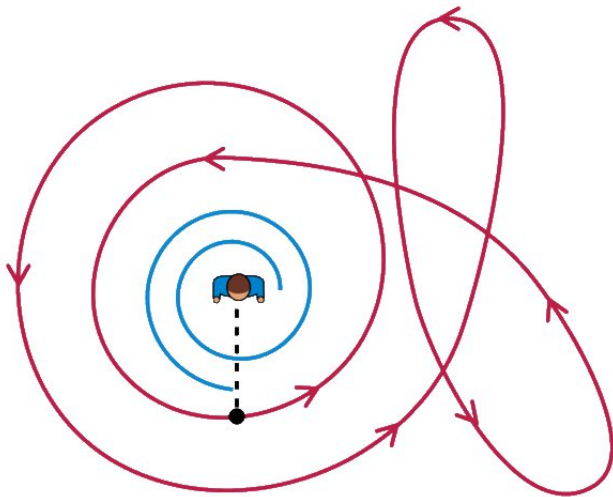
WINDING NUMBERS



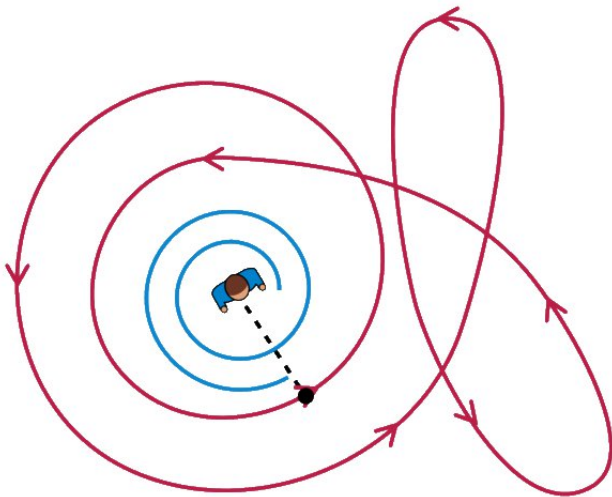
WINDING NUMBERS



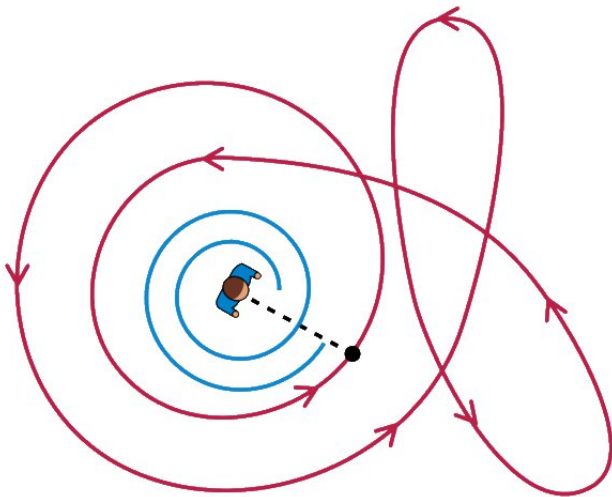
WINDING NUMBERS



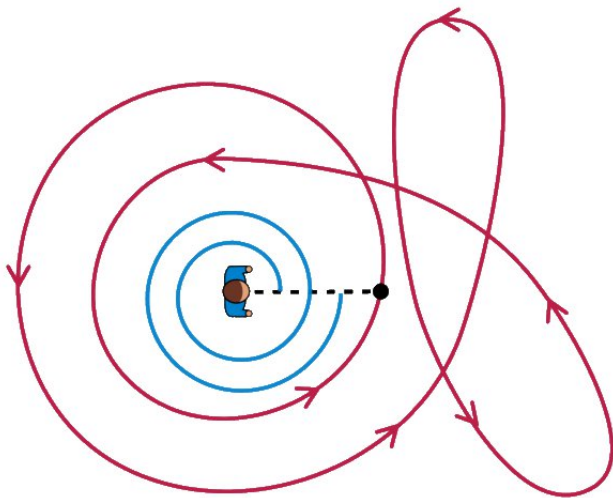
WINDING NUMBERS



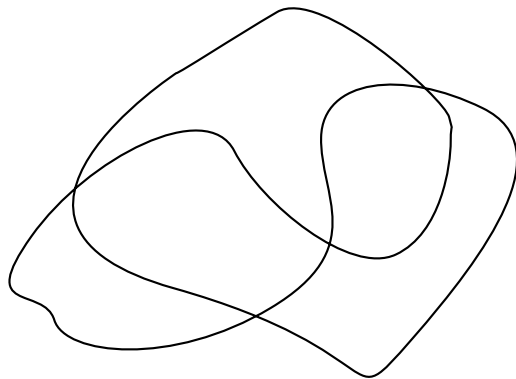
WINDING NUMBERS



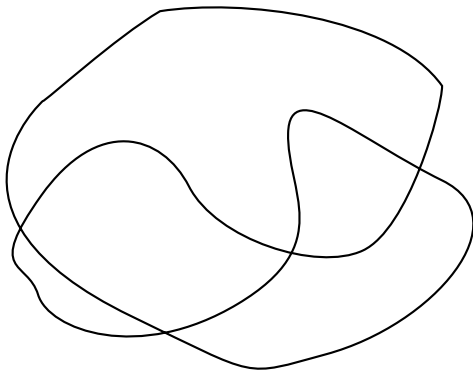
WINDING NUMBERS



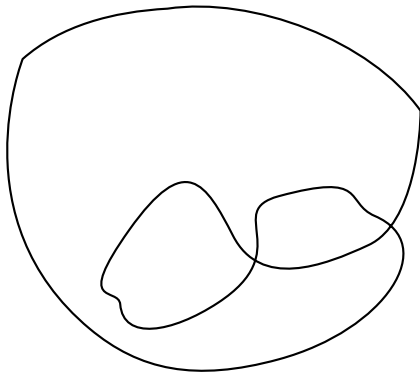
DEFORMING LOOPS



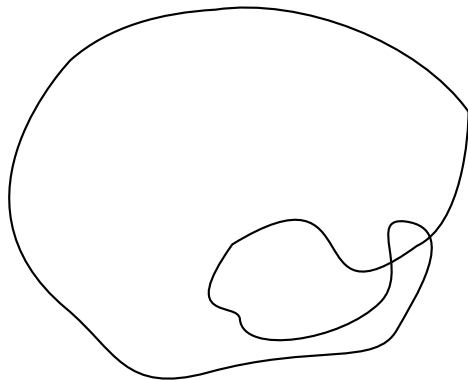
DEFORMING LOOPS



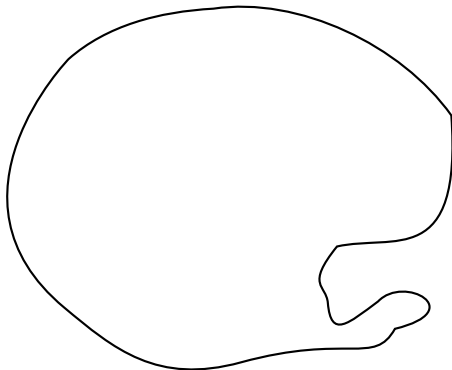
DEFORMING LOOPS



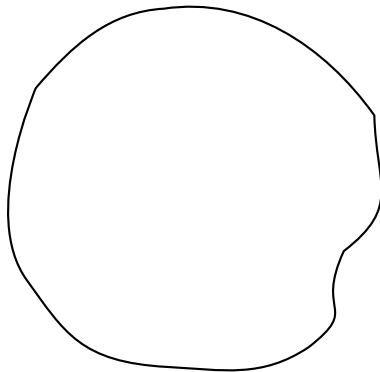
DEFORMING LOOPS



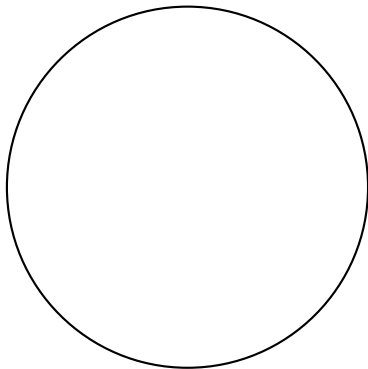
DEFORMING LOOPS



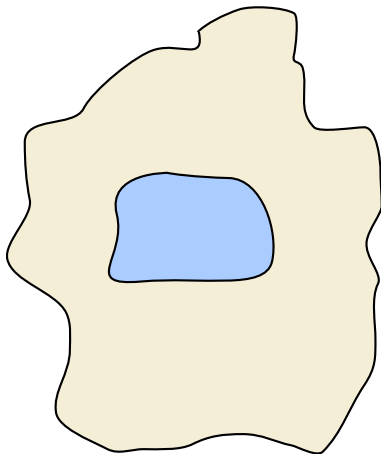
DEFORMING LOOPS



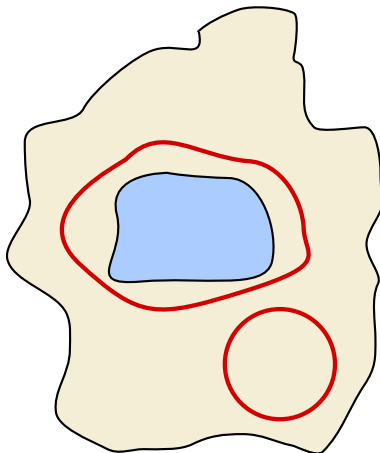
DEFORMING LOOPS



DEFORMING LOOP WITH OBSTACLES



DEFORMING LOOP WITH OBSTACLES



HOPF'S DEGREE THEOREM

HOPF'S DEGREE THEOREM

THEOREM.

HOPF'S DEGREE THEOREM

THEOREM. A loop γ_1

HOPF'S DEGREE THEOREM

THEOREM. A loop γ_1 can be deformed on to another loop γ_2

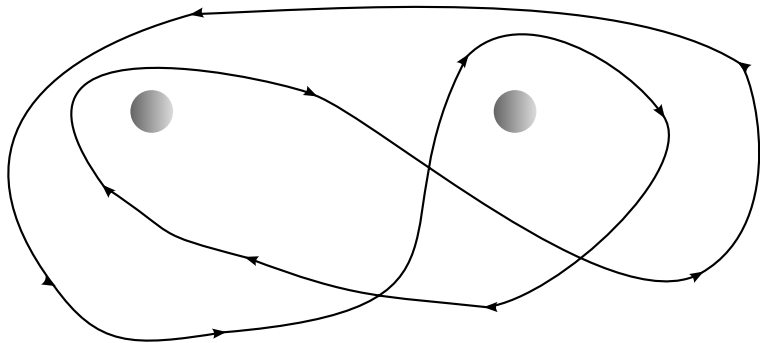
HOPF'S DEGREE THEOREM

THEOREM. A loop γ_1 can be deformed on to another loop γ_2 whilst avoiding the obstacle p

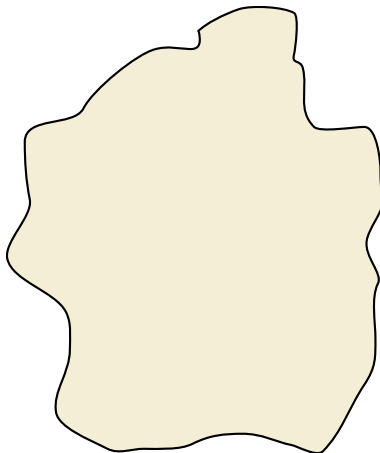
HOPF'S DEGREE THEOREM

THEOREM. A loop γ_1 can be deformed on to another loop γ_2 whilst avoiding the obstacle p if and only if γ_1 and γ_2 have the same winding number about p .

HANGING PROBLEM

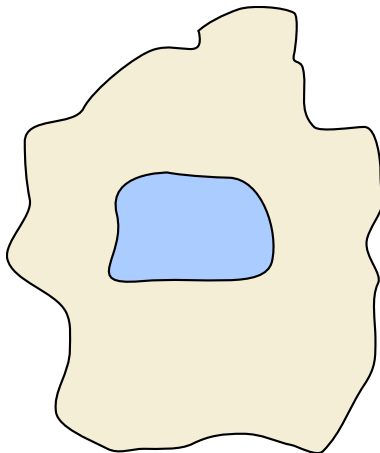


CONNECTIVITY

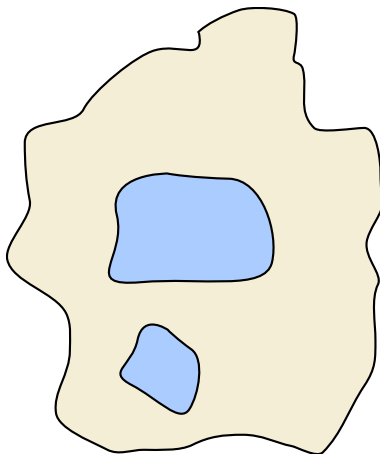


1

CONNECTIVITY

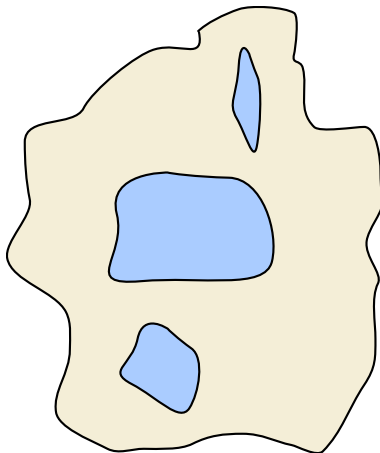


CONNECTIVITY

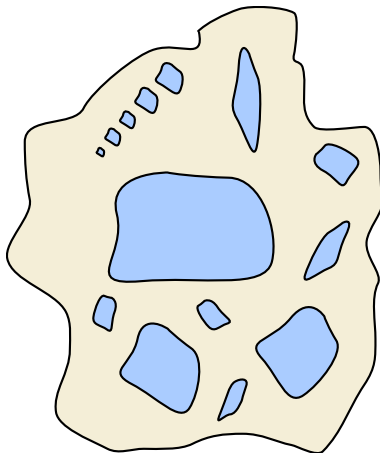


3

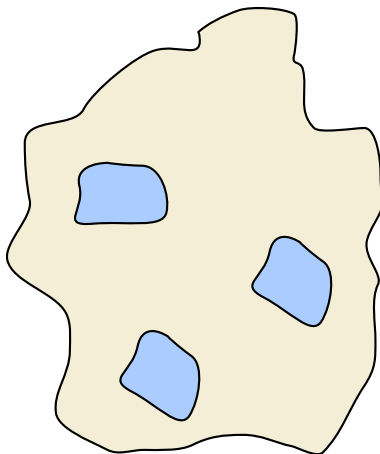
CONNECTIVITY



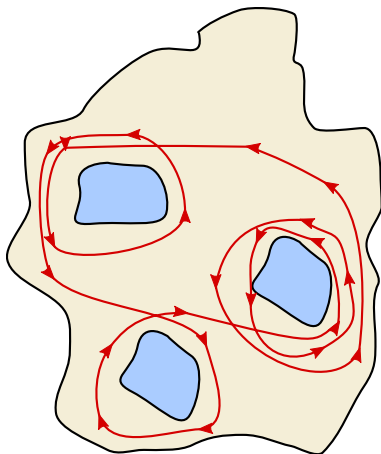
CONNECTIVITY

 ∞

CONNECTIVITY USING WINDING NUMBERS

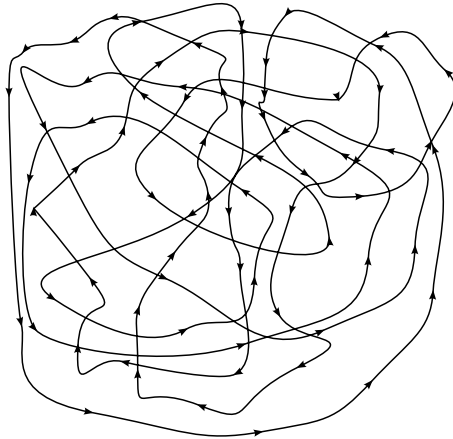
 $(2, -1, 3)$

CONNECTIVITY USING WINDING NUMBERS

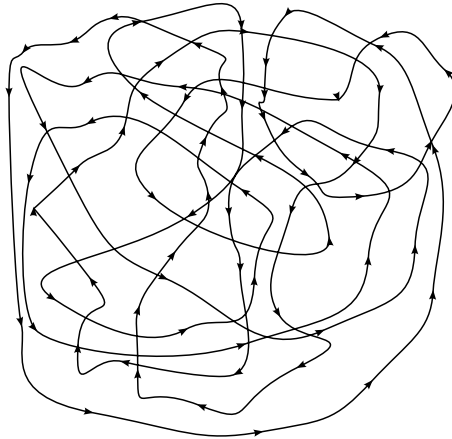


$(2, -1, 3)$

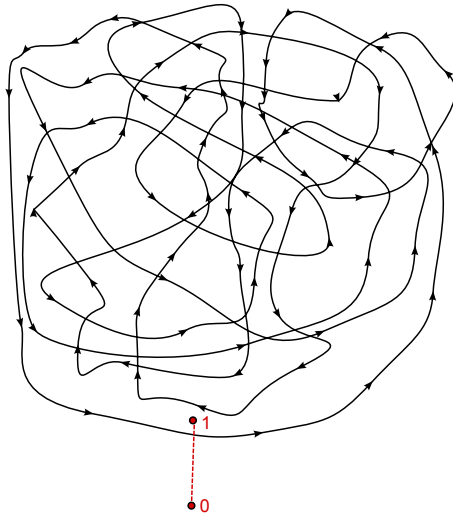
HOW TO CALCULATE WINDING NUMBERS



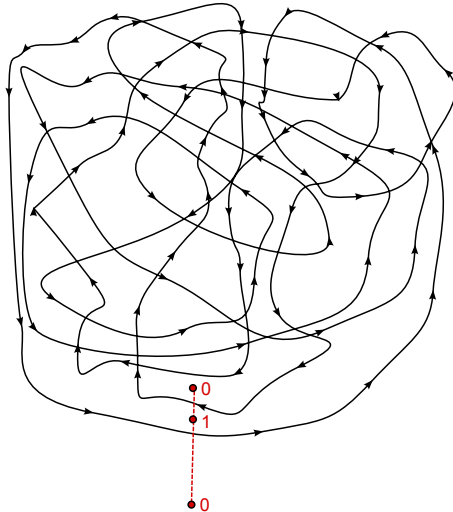
HOW TO CALCULATE WINDING NUMBERS



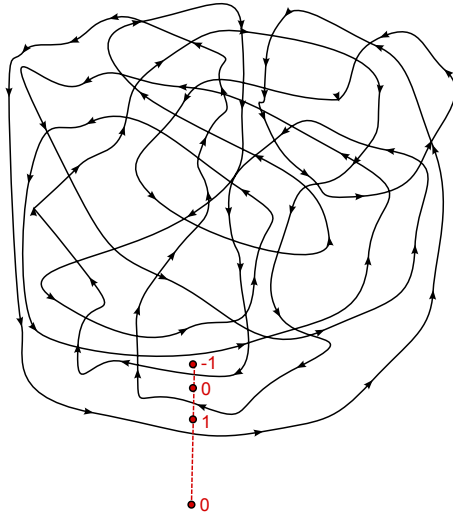
HOW TO CALCULATE WINDING NUMBERS



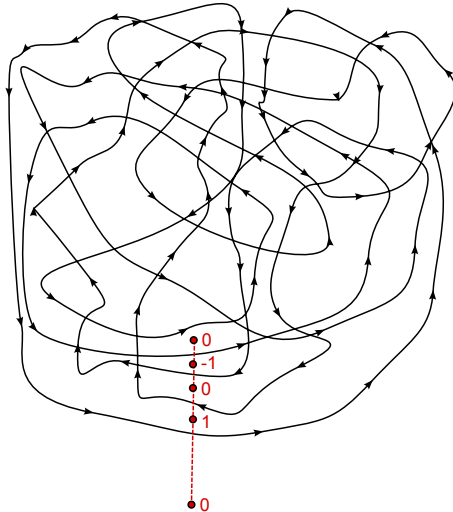
HOW TO CALCULATE WINDING NUMBERS



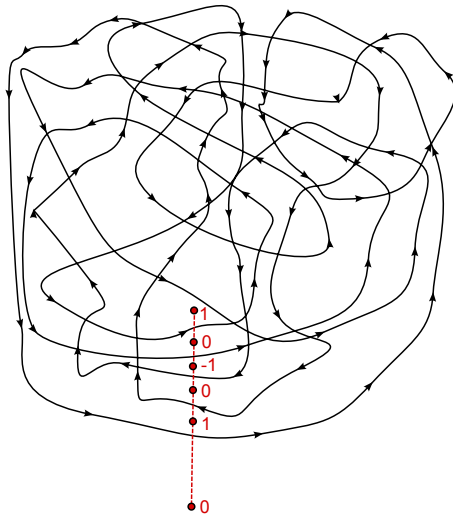
HOW TO CALCULATE WINDING NUMBERS



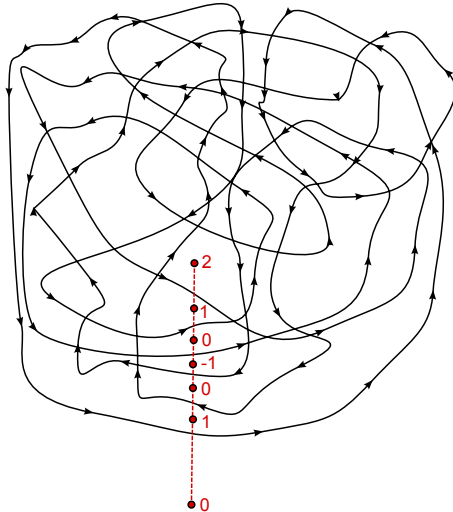
HOW TO CALCULATE WINDING NUMBERS



HOW TO CALCULATE WINDING NUMBERS

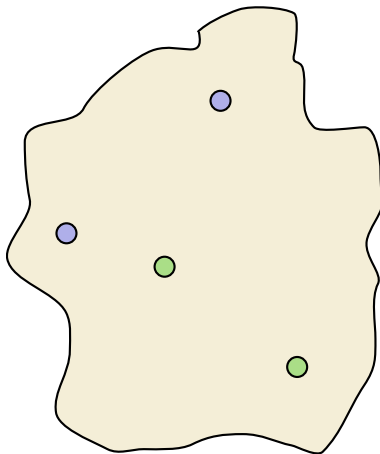


HOW TO CALCULATE WINDING NUMBERS

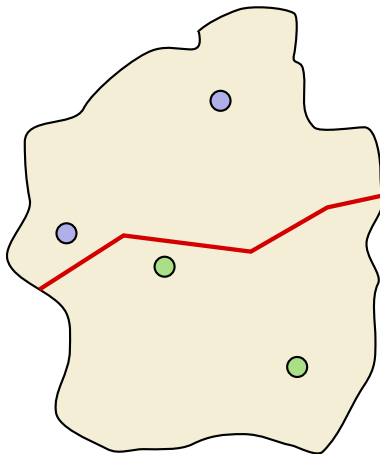


DIVIDING WALLS

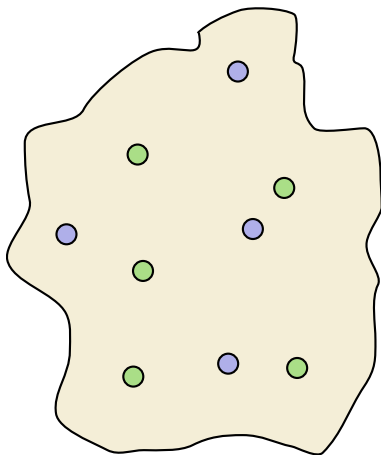
DIVIDING WALLS



DIVIDING WALLS



PROBLEM

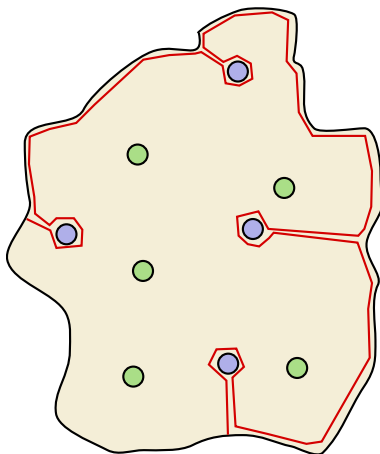


Is there a dividing wall for this set of towns?

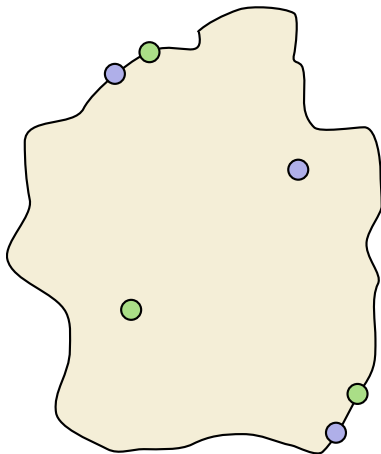
Experiment with other configurations of towns.

A GENERAL STRATEGY

A GENERAL STRATEGY

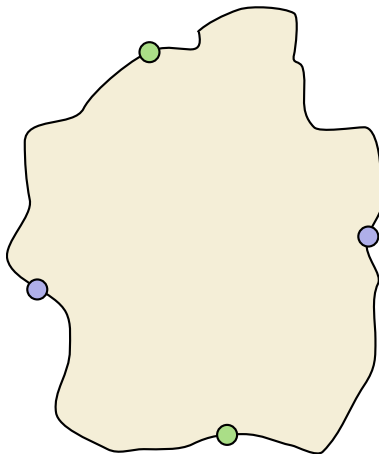


COASTAL TOWNS

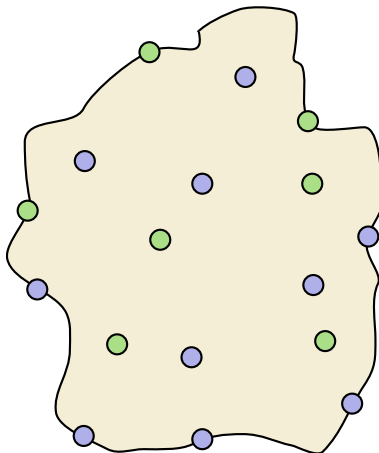


Experiment with other configurations of coastal towns
and inland towns.

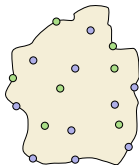
SIMPLIST PROBLEMATIC CONFIGURATION



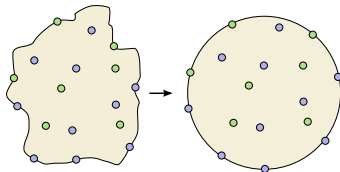
DISCARD THE PROBLEM CASE



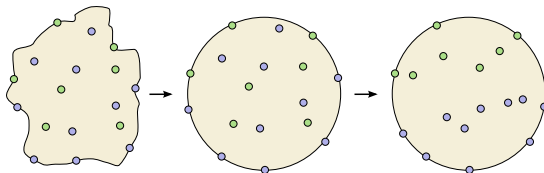
THINK TOPOLOGICALLY



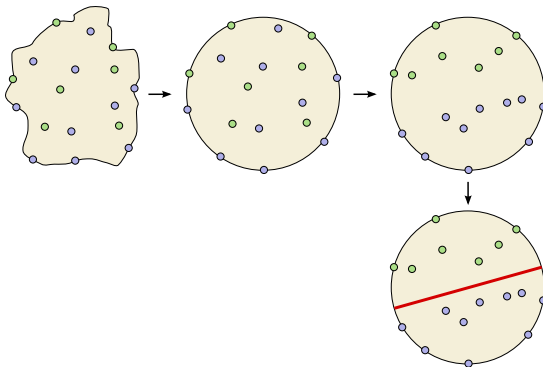
THINK TOPOLOGICALLY



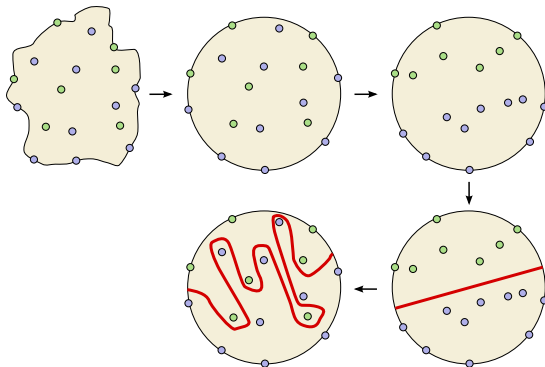
THINK TOPOLOGICALLY



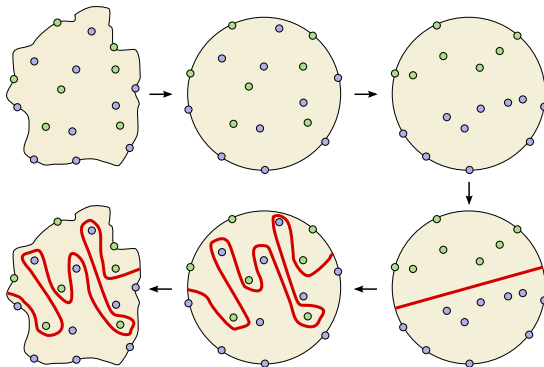
THINK TOPOLOGICALLY



THINK TOPOLOGICALLY



THINK TOPOLOGICALLY



DIVIDING WALLS THEOREM

DIVIDING WALLS THEOREM

THEOREM.

DIVIDING WALLS THEOREM

THEOREM. A collection of blue and green towns

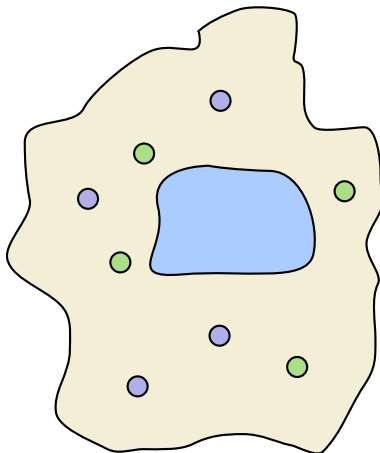
DIVIDING WALLS THEOREM

THEOREM. A collection of blue and green towns can be separated (blue from green) by a dividing wall

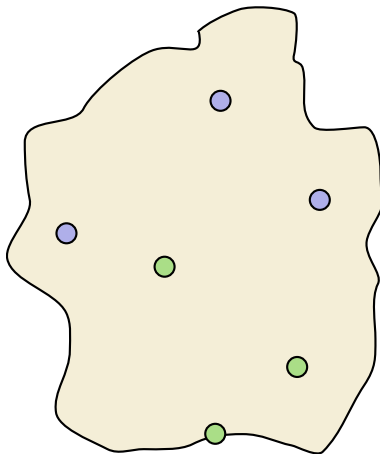
DIVIDING WALLS THEOREM

THEOREM. A collection of blue and green towns can be separated (blue from green) by a dividing wall if and only if there are not four coastal towns configured in the same way as the simplest problematic example.

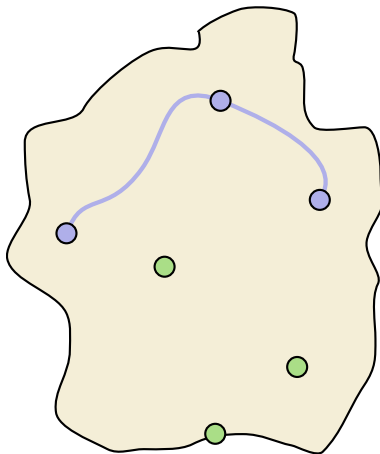
LAKES



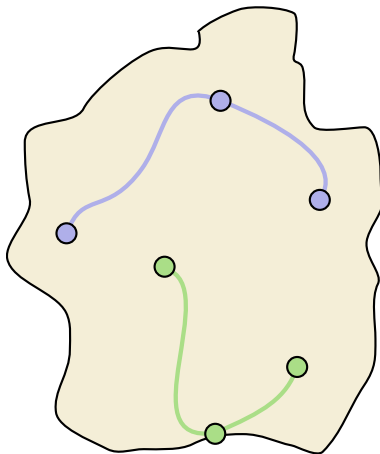
CONNECTING ROADS



CONNECTING ROADS



CONNECTING ROADS



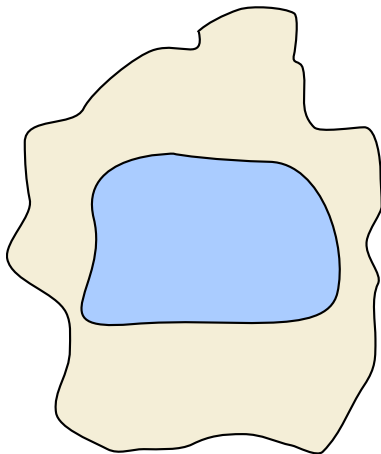
Investigate dividing walls and connecting roads for three town colours.

EXOTIC ISLANDS

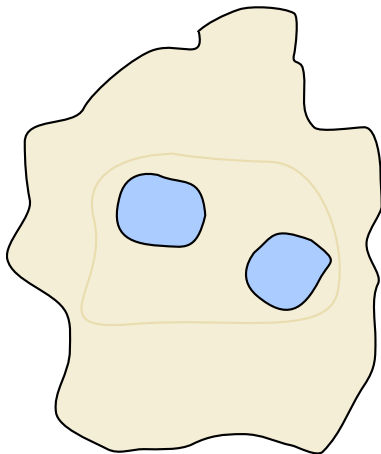
FRIEDRICH SCHOTTKY (1851–1935)



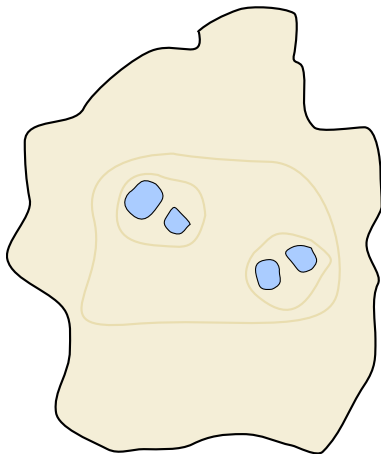
SCHOTTKY ISLAND



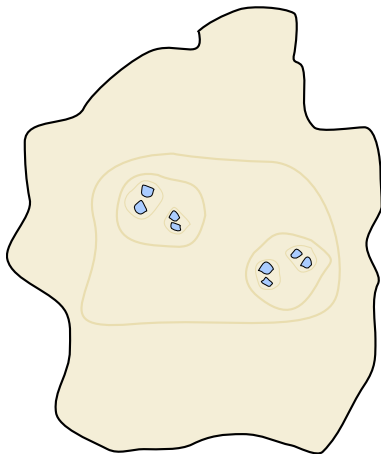
SCHOTTKY ISLAND



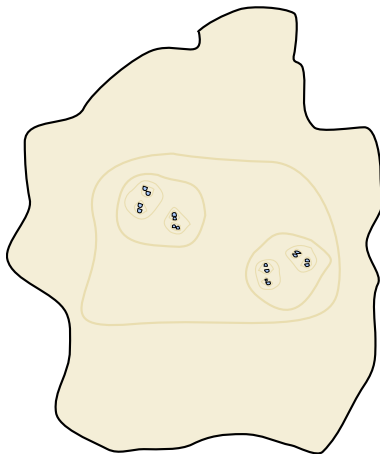
SCHOTTKY ISLAND



SCHOTTKY ISLAND



SCHOTTKY ISLAND



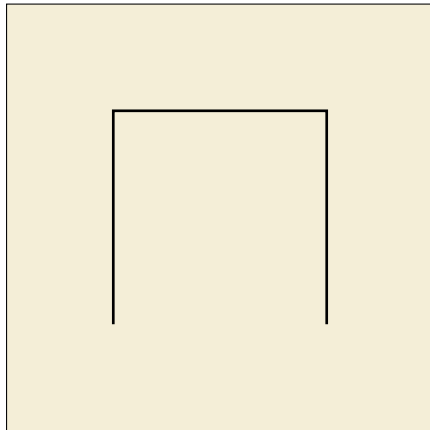
GIUSEPPE PEANO (1858–1932)



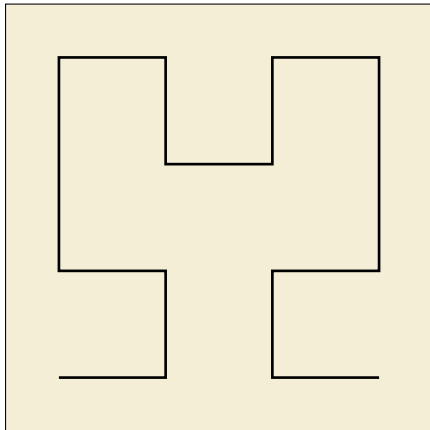
PEANO ISLAND



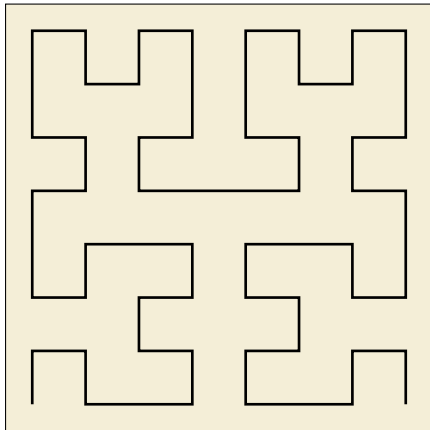
PEANO ISLAND



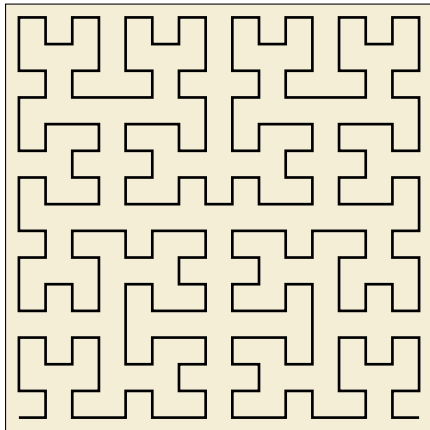
PEANO ISLAND



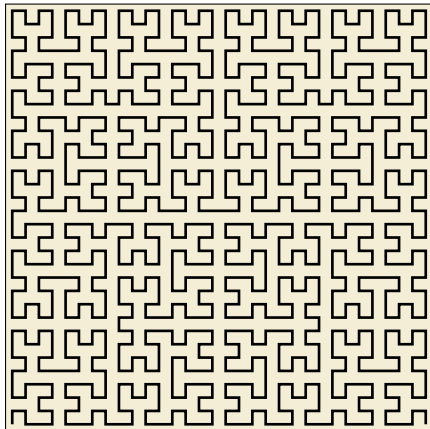
PEANO ISLAND



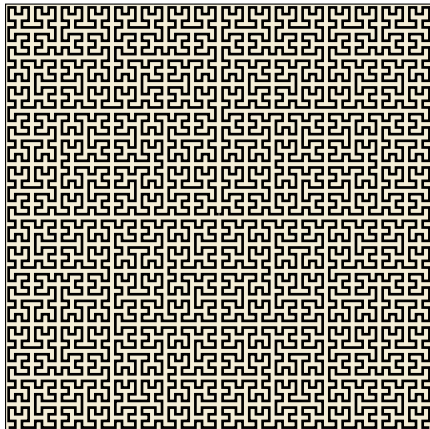
PEANO ISLAND



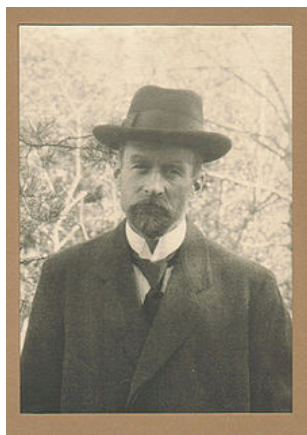
PEANO ISLAND



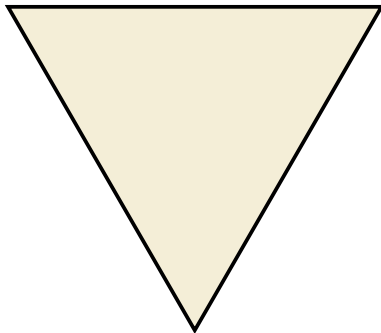
PEANO ISLAND



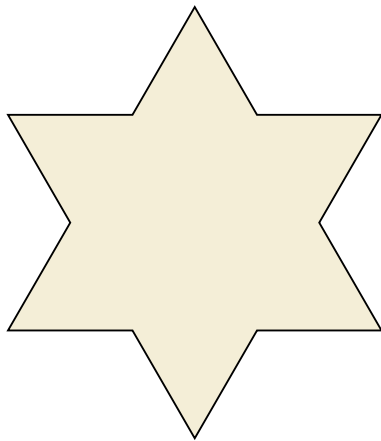
HELGE VON KOCH (1870–1924)



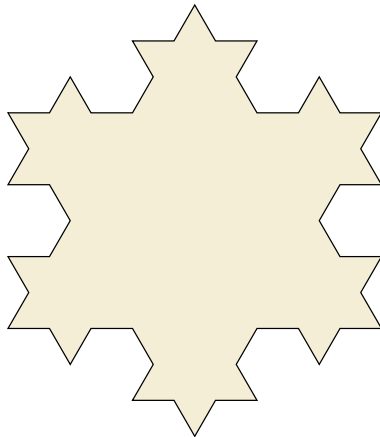
VON KOCH ISLAND



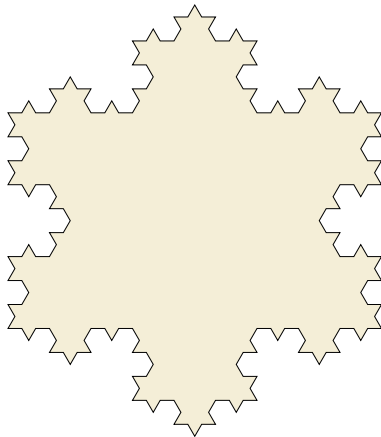
VON KOCH ISLAND



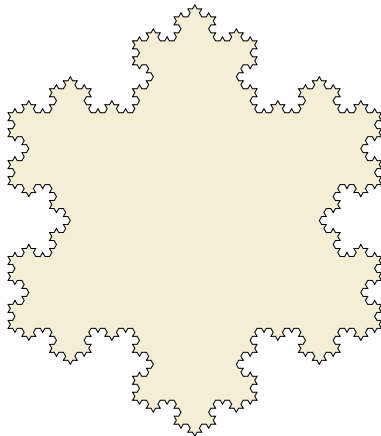
VON KOCH ISLAND



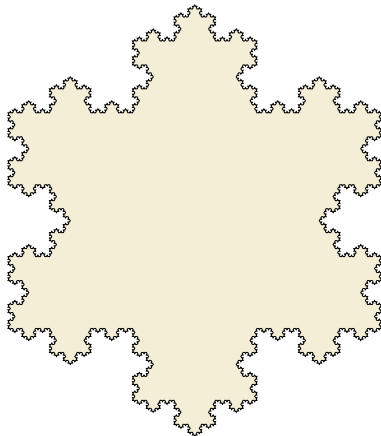
VON KOCH ISLAND



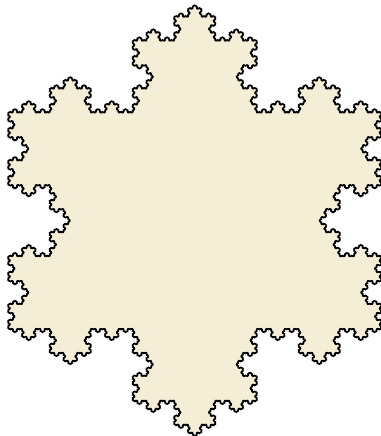
VON KOCH ISLAND



VON KOCH ISLAND



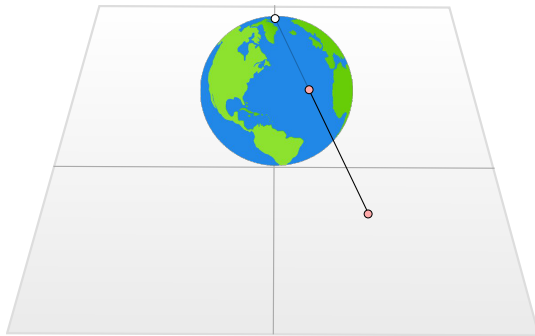
VON KOCH ISLAND



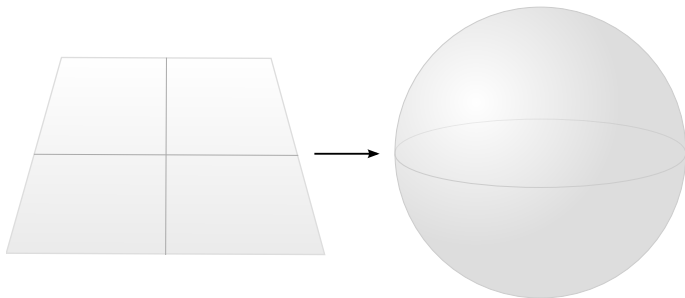
BERNHARD RIEMANN (1826–1866)



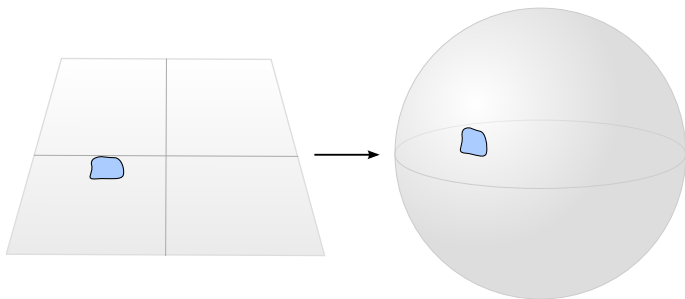
STEREOGRAPHIC PROJECTION



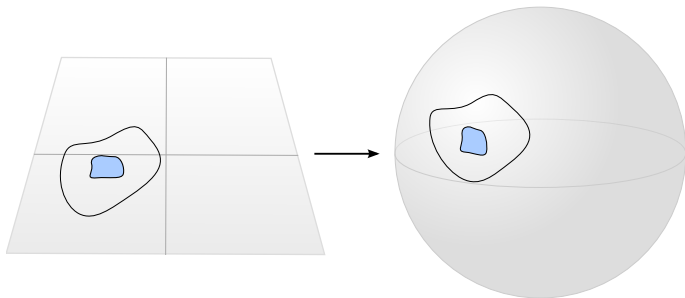
TOPOLOGICAL DISTINCTION



TOPOLOGICAL DISTINCTION



TOPOLOGICAL DISTINCTION



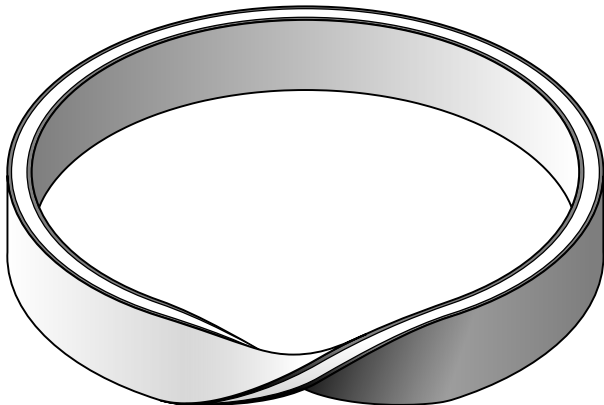
AUGUST MÖBIUS (1790–1868)



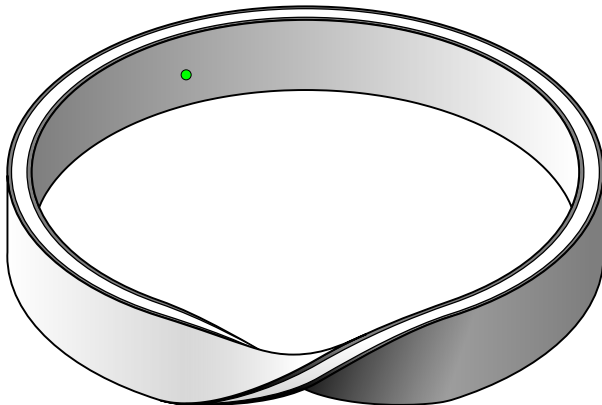
Ad. F. Möbius del.

A. F. Möbius.

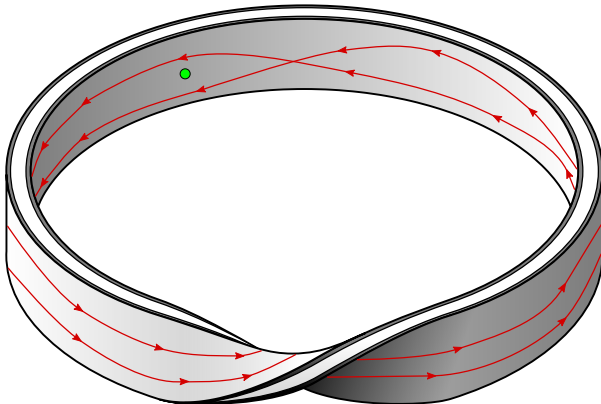
MÖBIUS STRIP



MÖBIUS STRIP



MÖBIUS STRIP



THANK YOU!