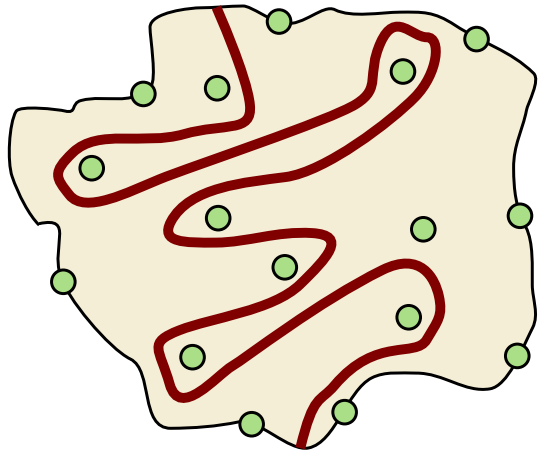
Geometry, topology, and topography

Ian Short



http://maths.org/~ims25/maths/presentations/Bletchley2011.pdf

Geometry

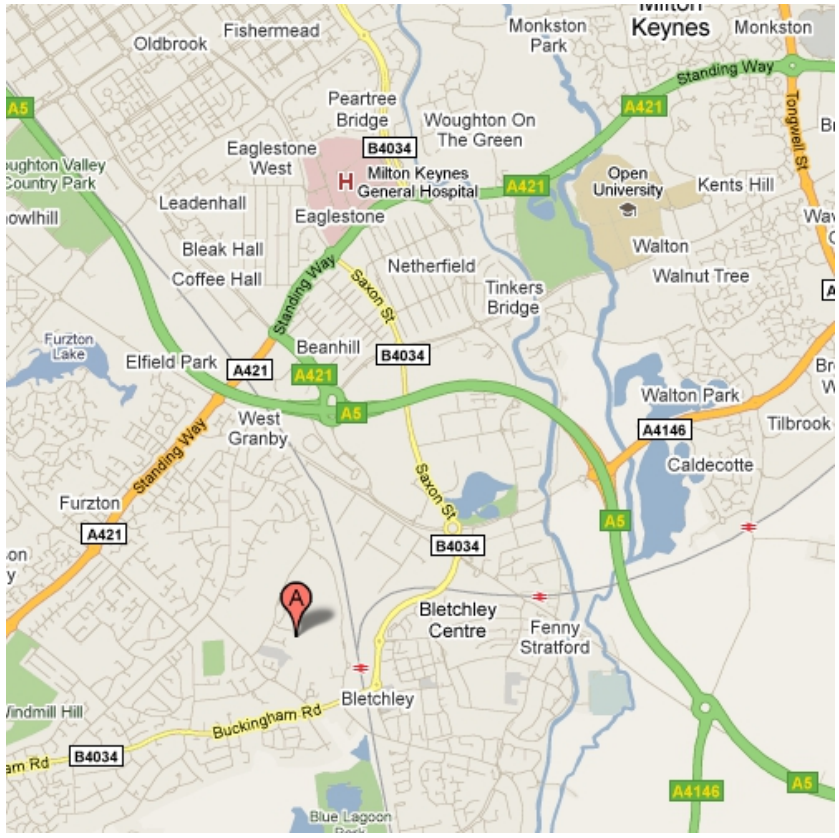
What is topography?

Topography

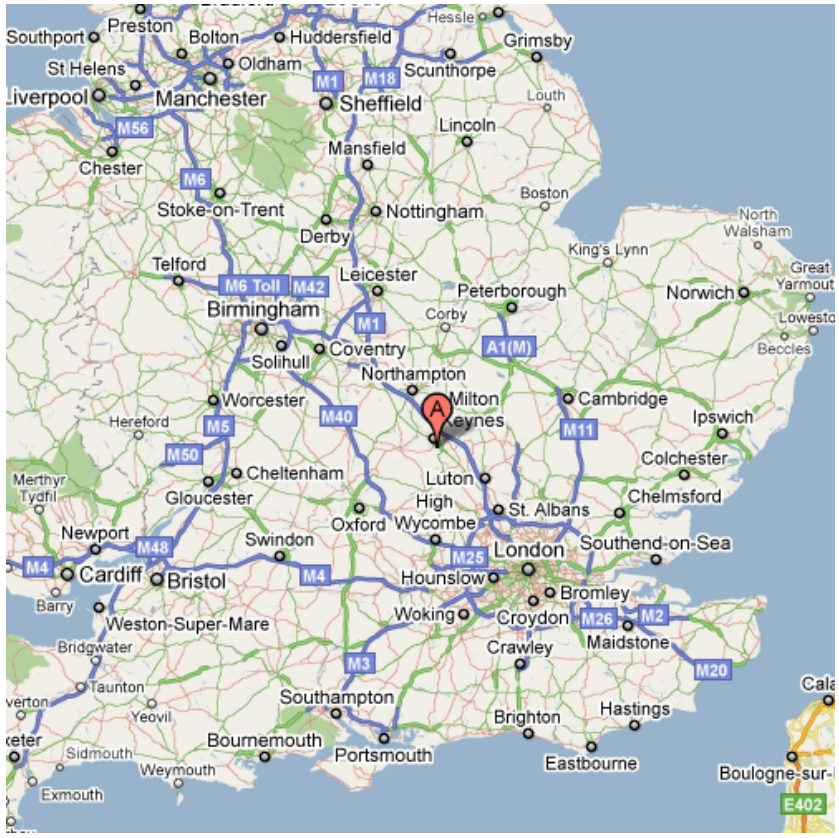
Topography


NORTH CAPE
Round
Barrel
Blue Mountain
Pond
Stone
Dry Heights
Wreck
Tomb
Crow Peak
Temple
Remains
Scar
Mountain
Pirate
Point
Blue Bay
EAST CAPE
Kraken
Count
Bay
Costa's Rock
Manmade Hole
Curl
Rock
Hanging
Bats
Gate
Rocks
Slough of 3
4 Trees
Blue
Tree
Point
Blue
Deep
Hole
One
Stone
Witch Circle
Hollow Trees
Six Islands
Bay of Bebai
Creek
Dunes
Short Peak
Red
Rock
White Sands
Shrubs
Serpent
HALLOWS
Horse Ba

Sense of direction

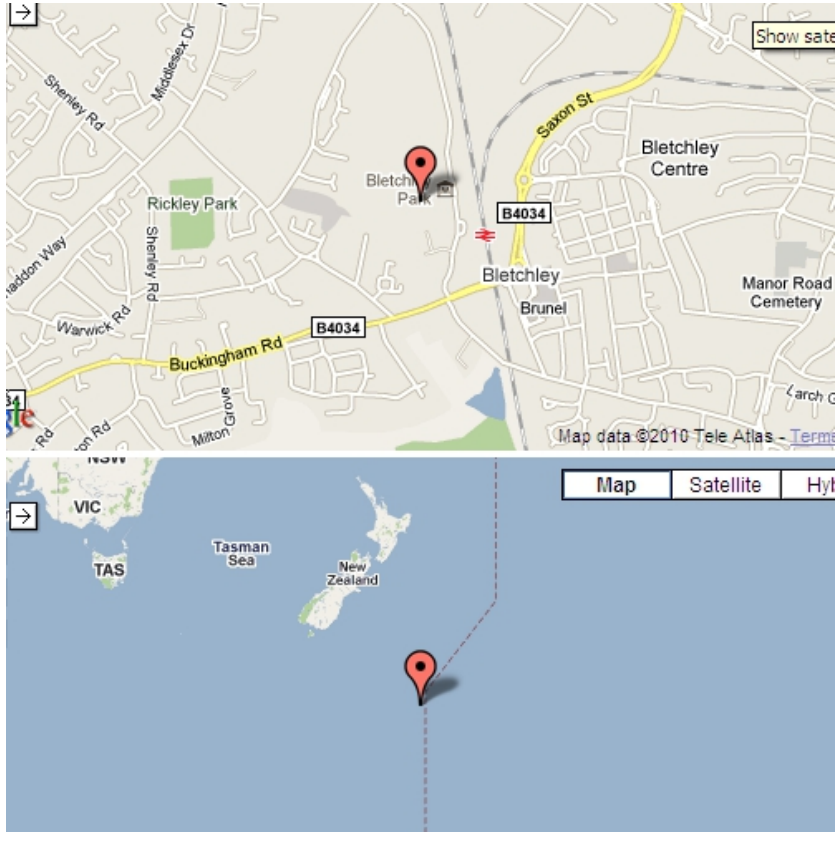


Sense of direction



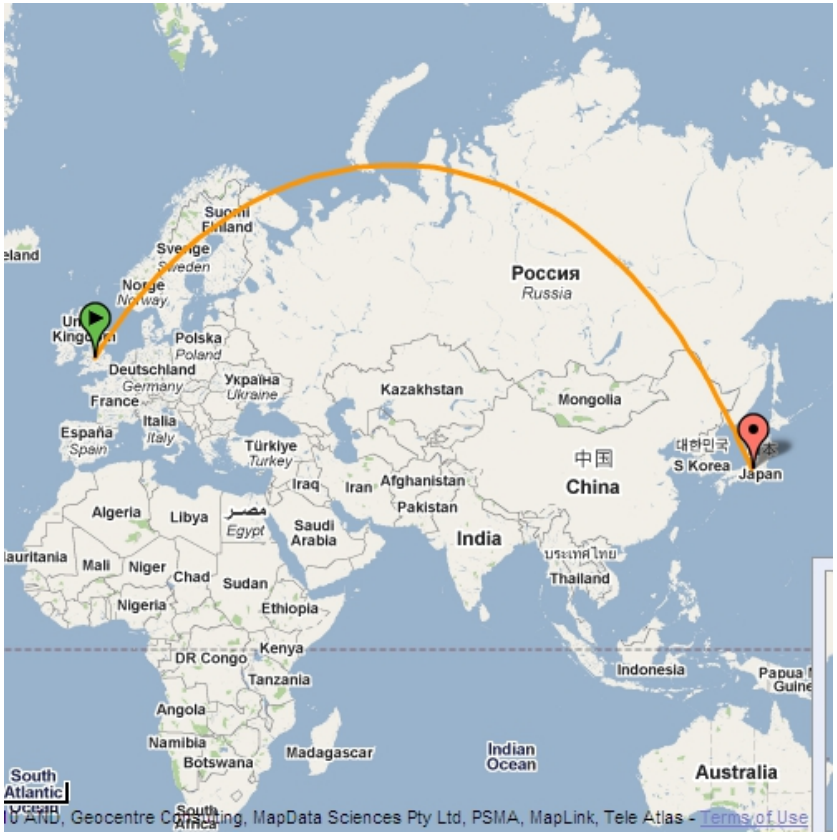
Sense of direction

Antipodal points



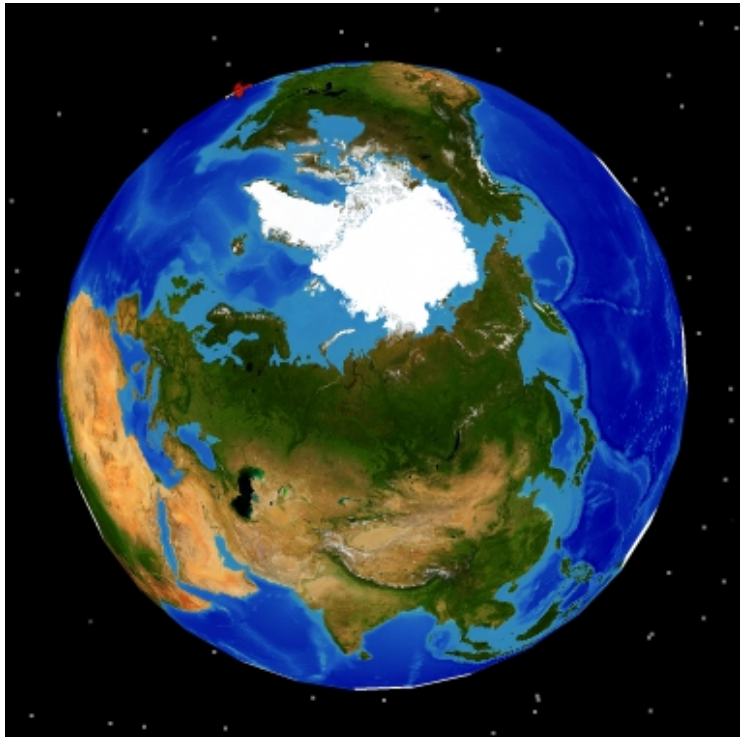
http://www.antipodemap.com/

Great circles



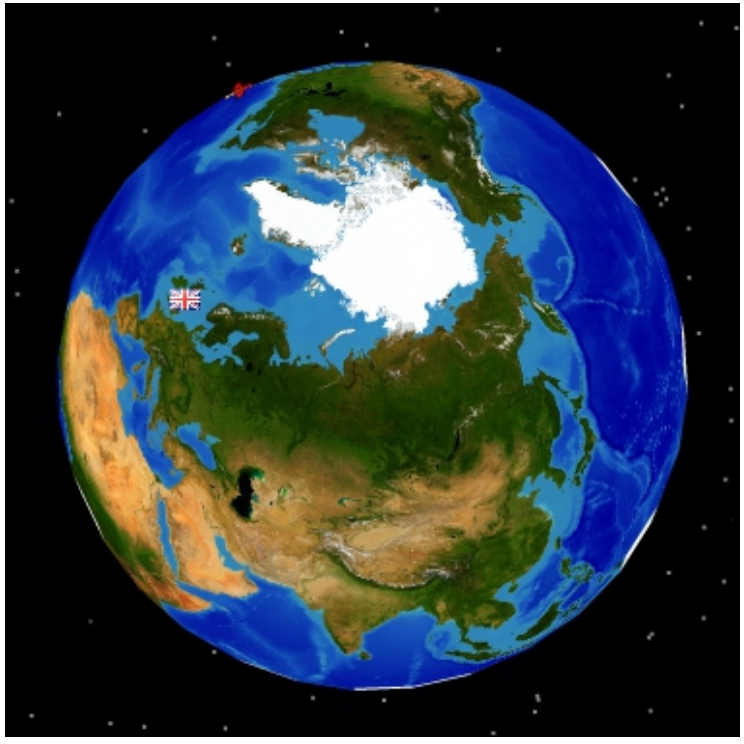
http://maps.forum.nu/gm_flight_path.html

Great circles



http://www.earthbrowser.com/

Great circles



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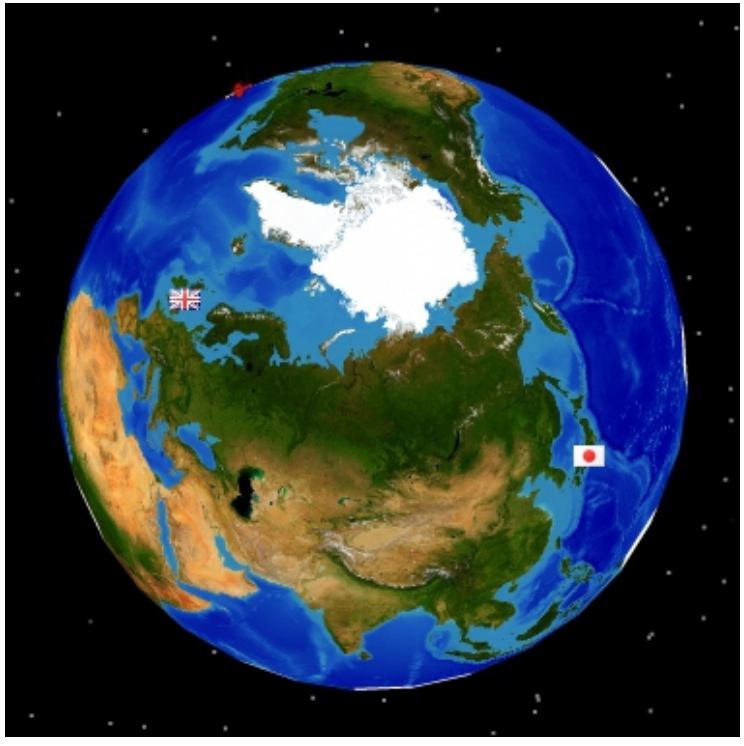
Great circles



http://www.earthbrowser.com/

Great circles

http://www.earthbrowser.com/

Great circles

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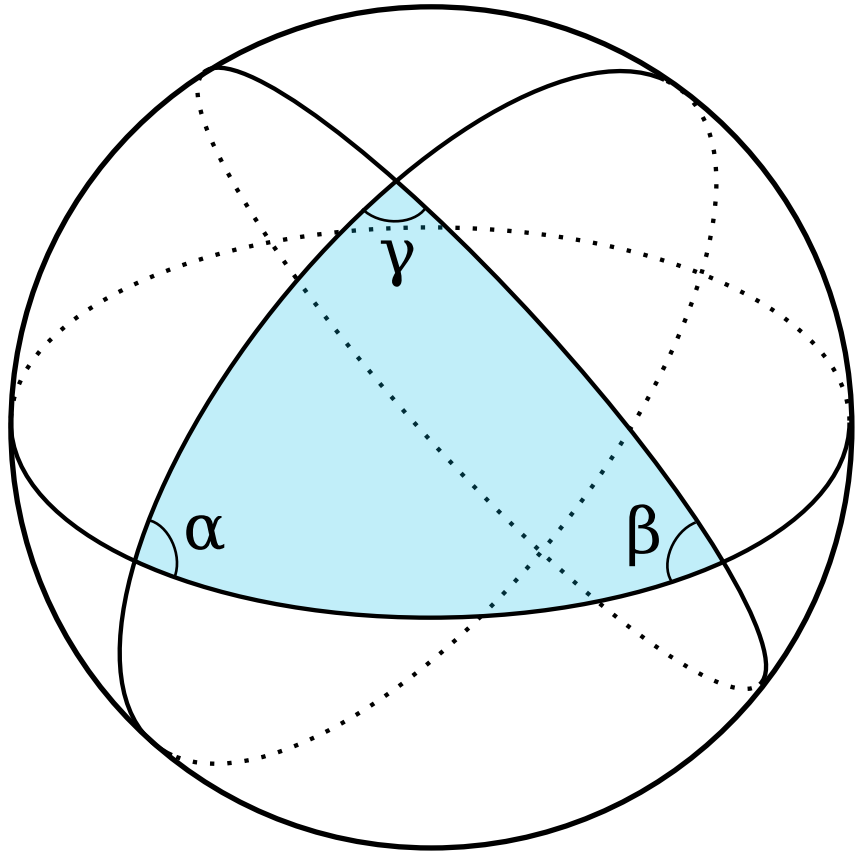
Great circles

http://www.earthbrowser.com/

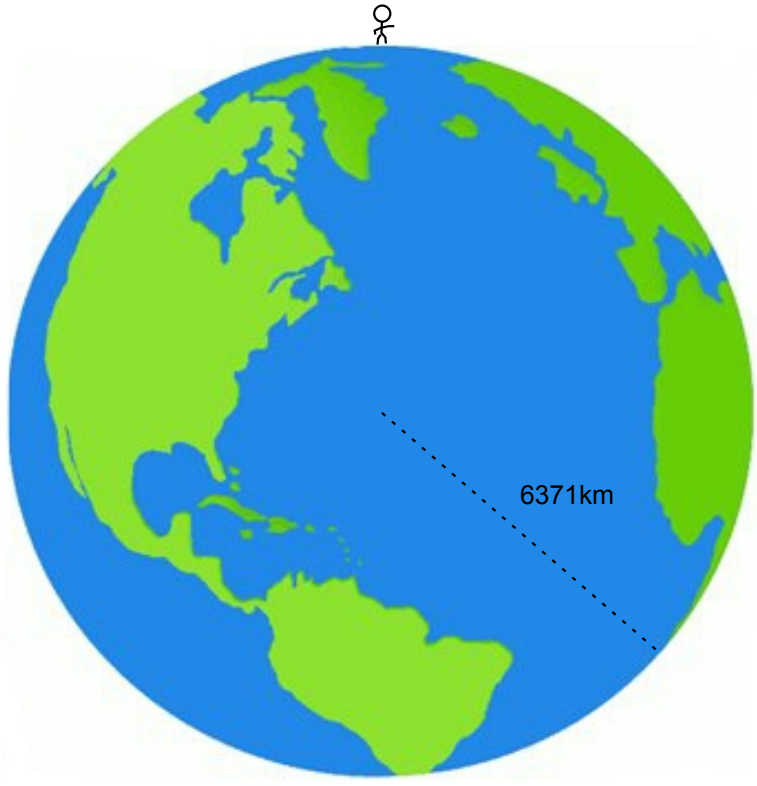
Compass directions

Spherical triangles

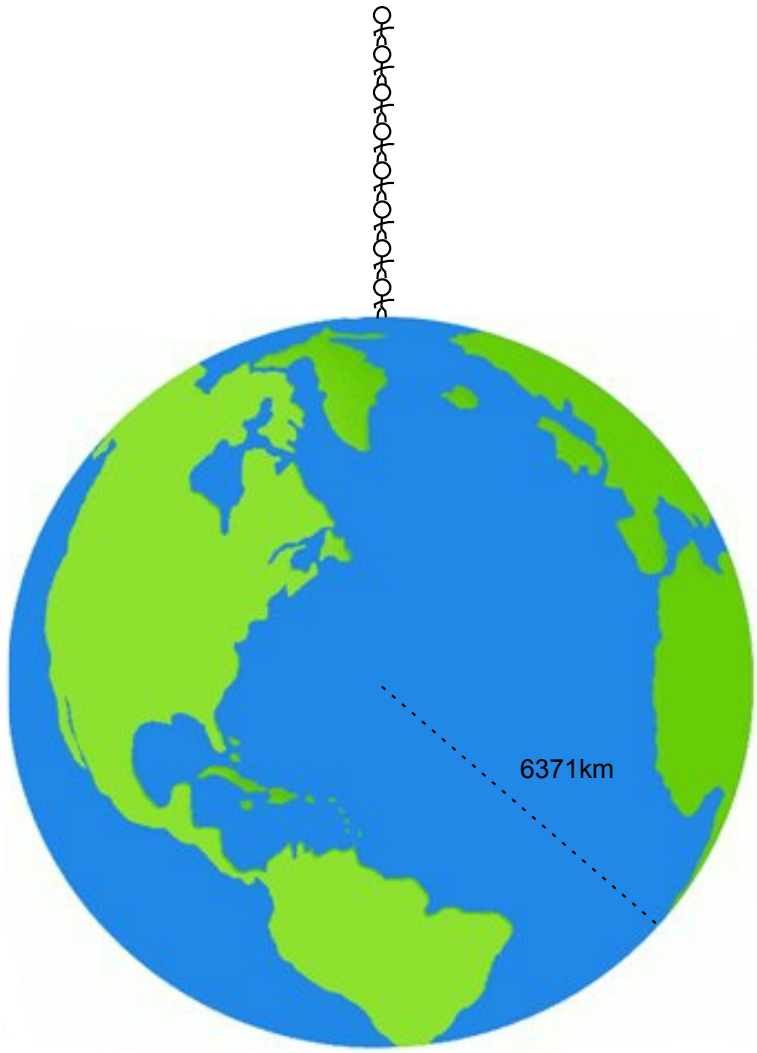


Mouse problem

How far to the horizon?



How far to the horizon?



Topology

What is topology?

Topology

No straight lines...

Topology

No straight lines...

No angles...

Topology

No straight lines...

No angles...

No distances...

Topology

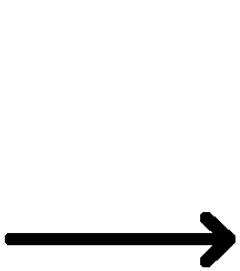
No straight lines...

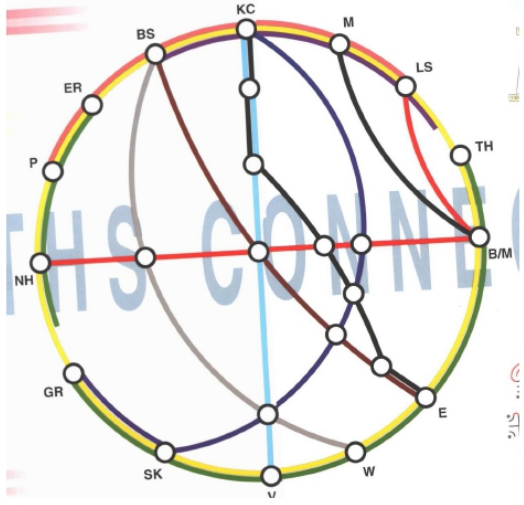
No angles...

No distances...

Just *shapes* and *distortion* of shapes.

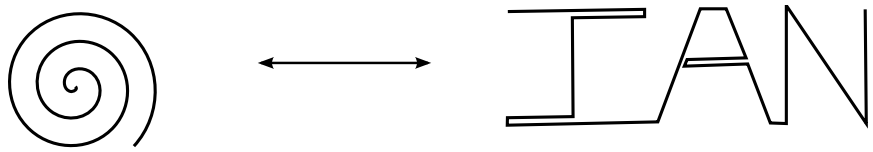
From *Get knotted!*





Neon signs

Dimensions



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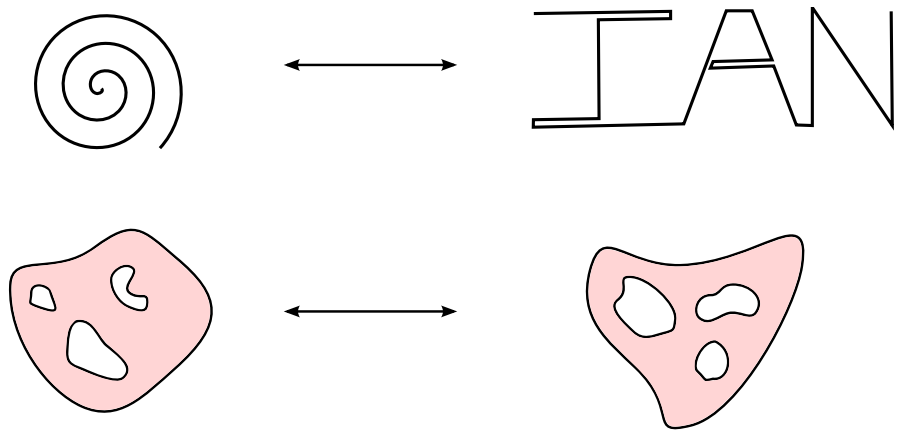
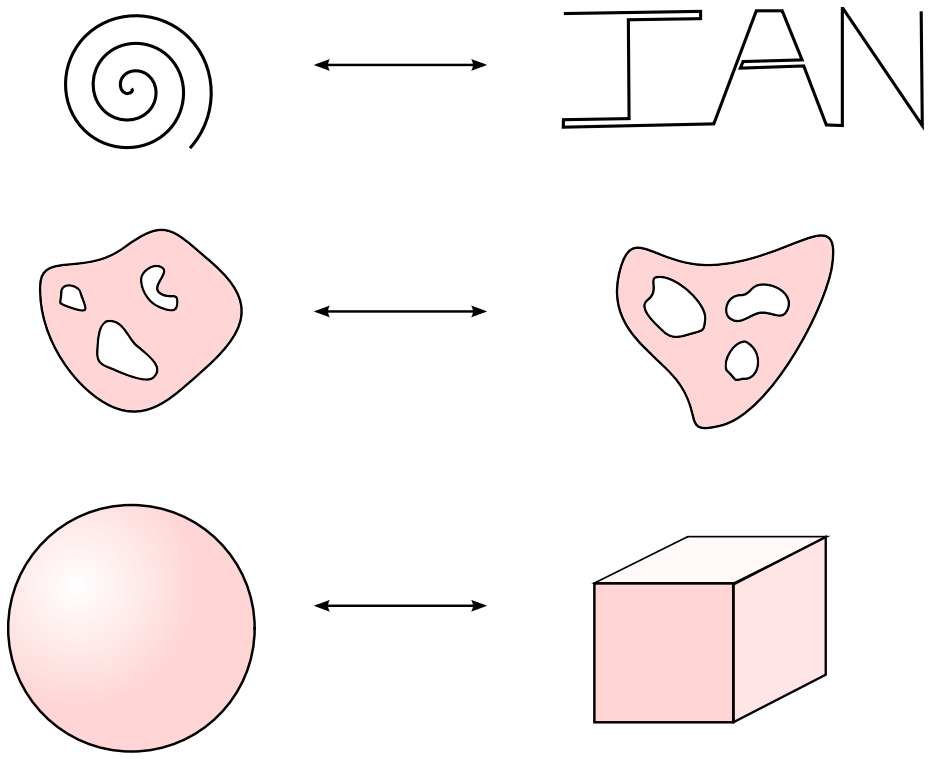
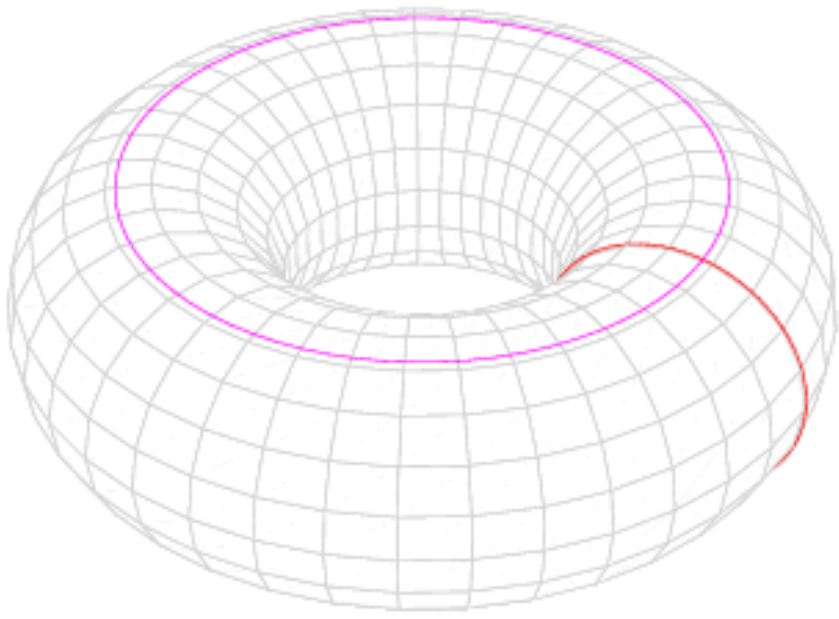


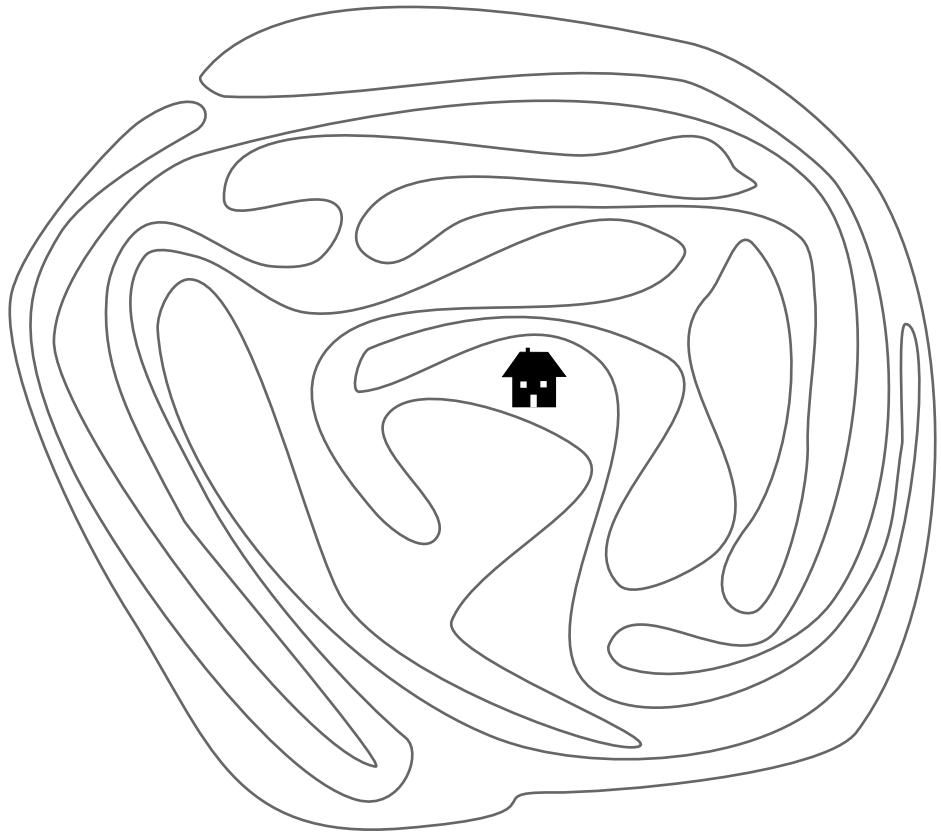
Dimensions



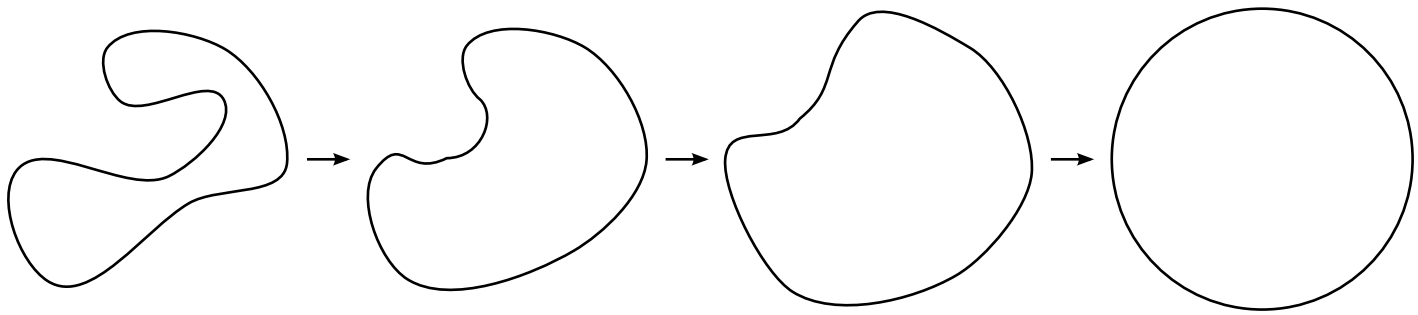
Torus



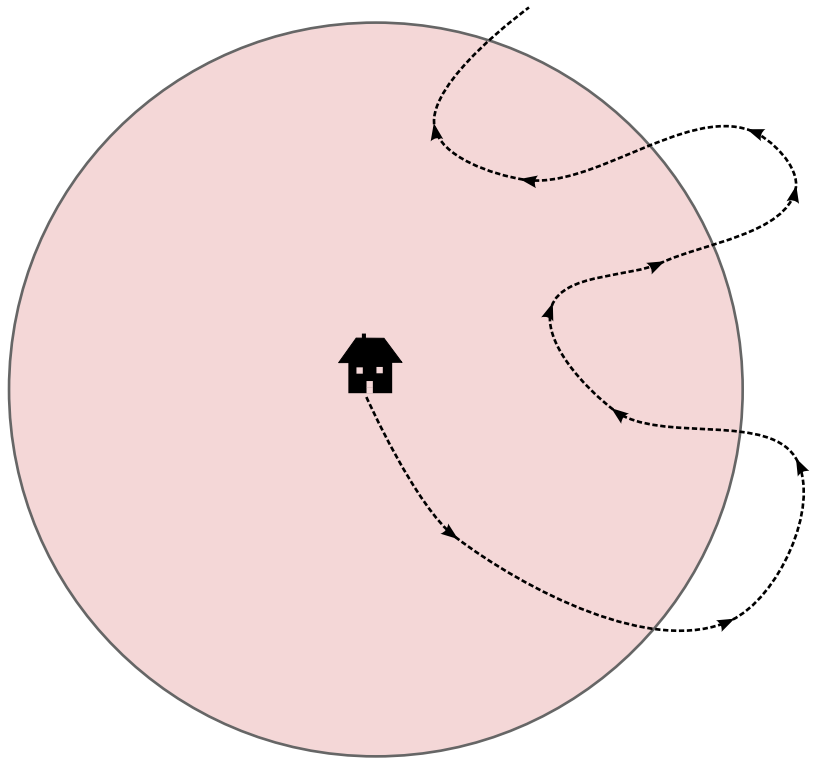
Unreliable builders



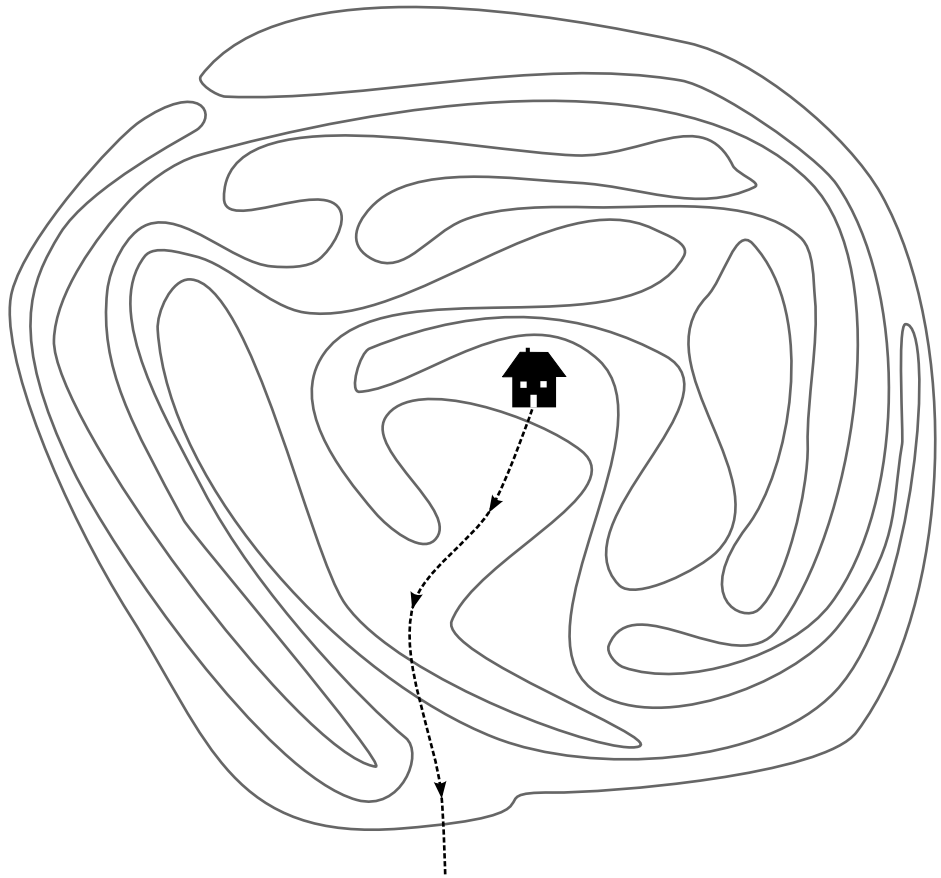
Continuous distortion



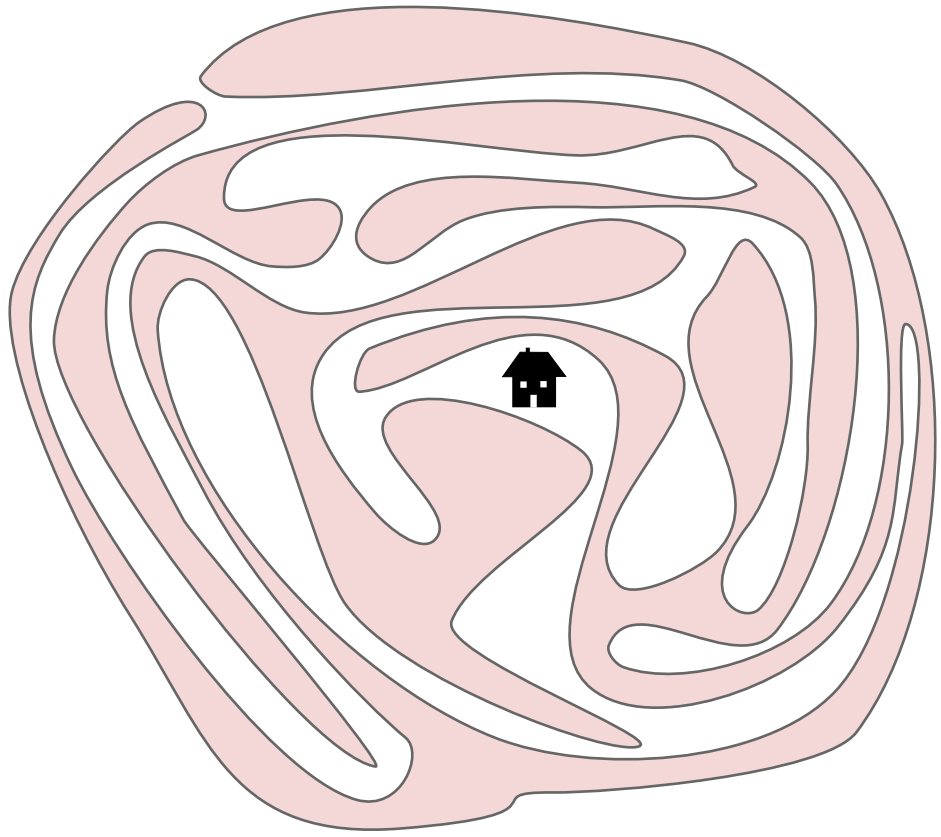
Count crossings



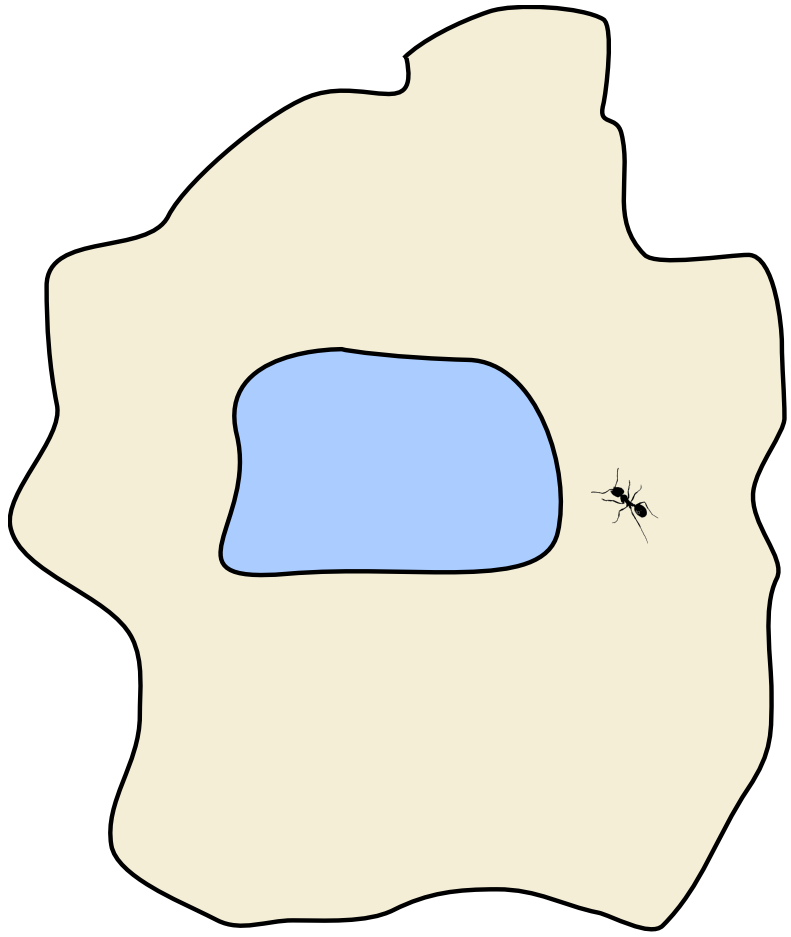
Count crossings



Fill

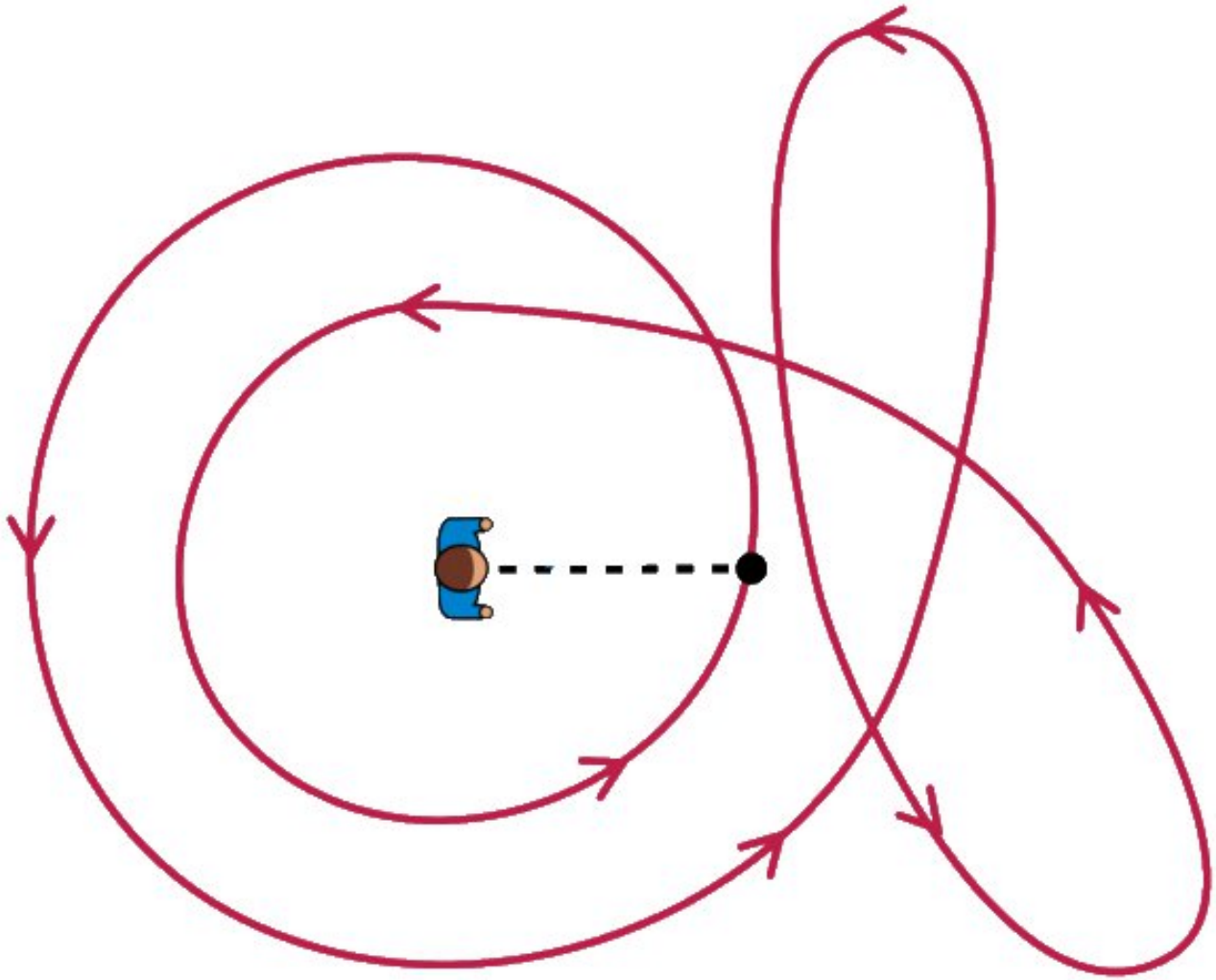


Lake or sea?



Hanging problem

Winding numbers



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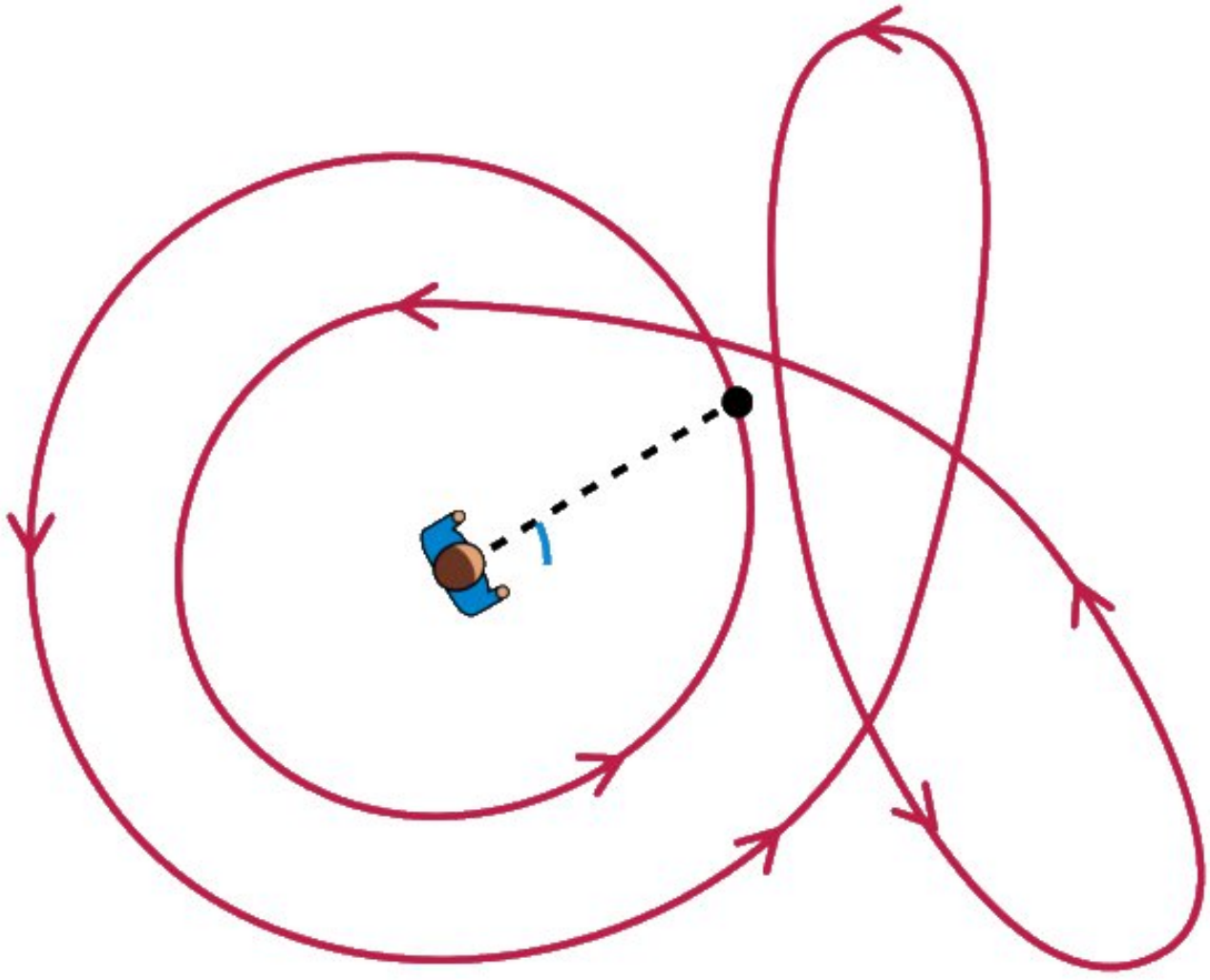

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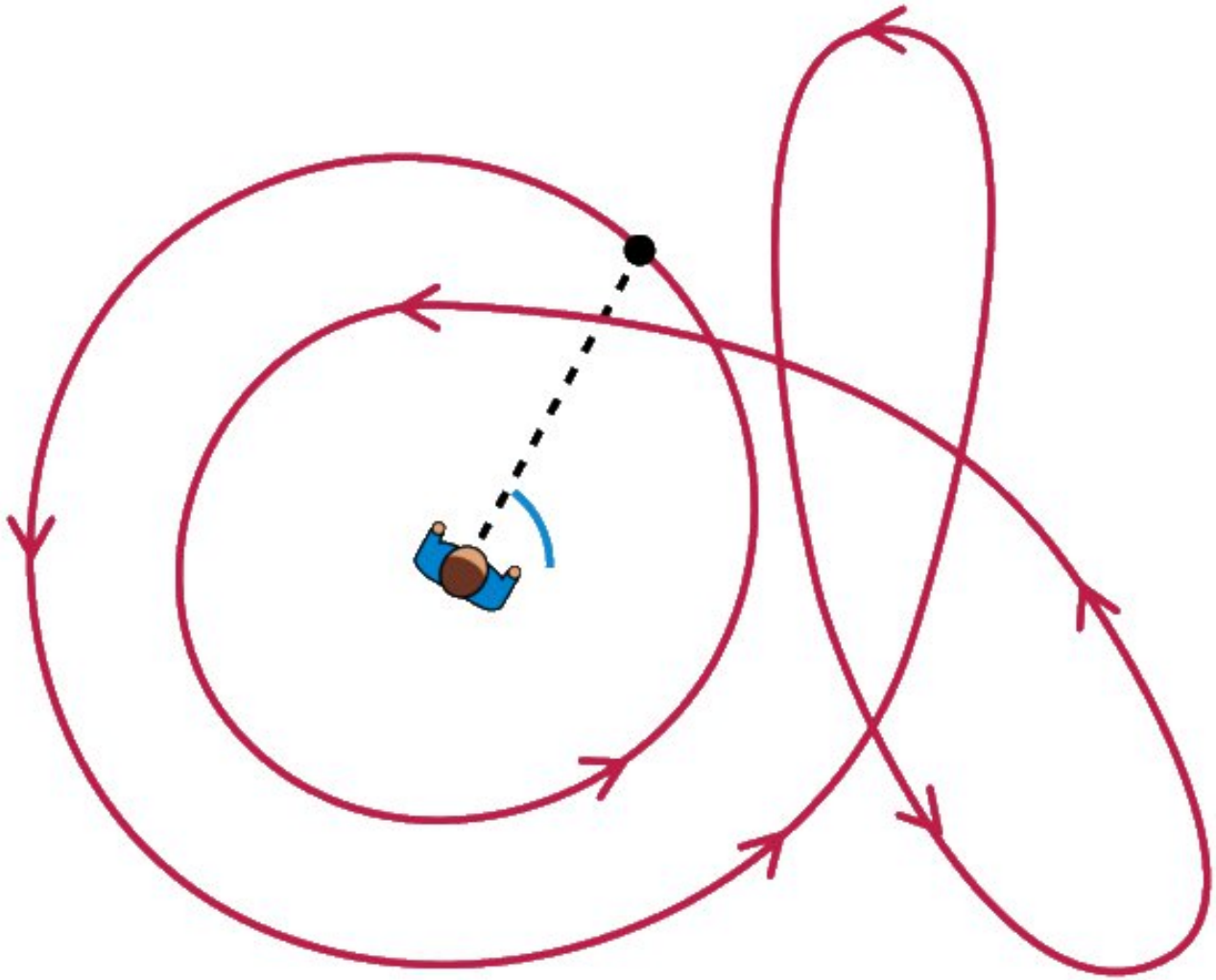

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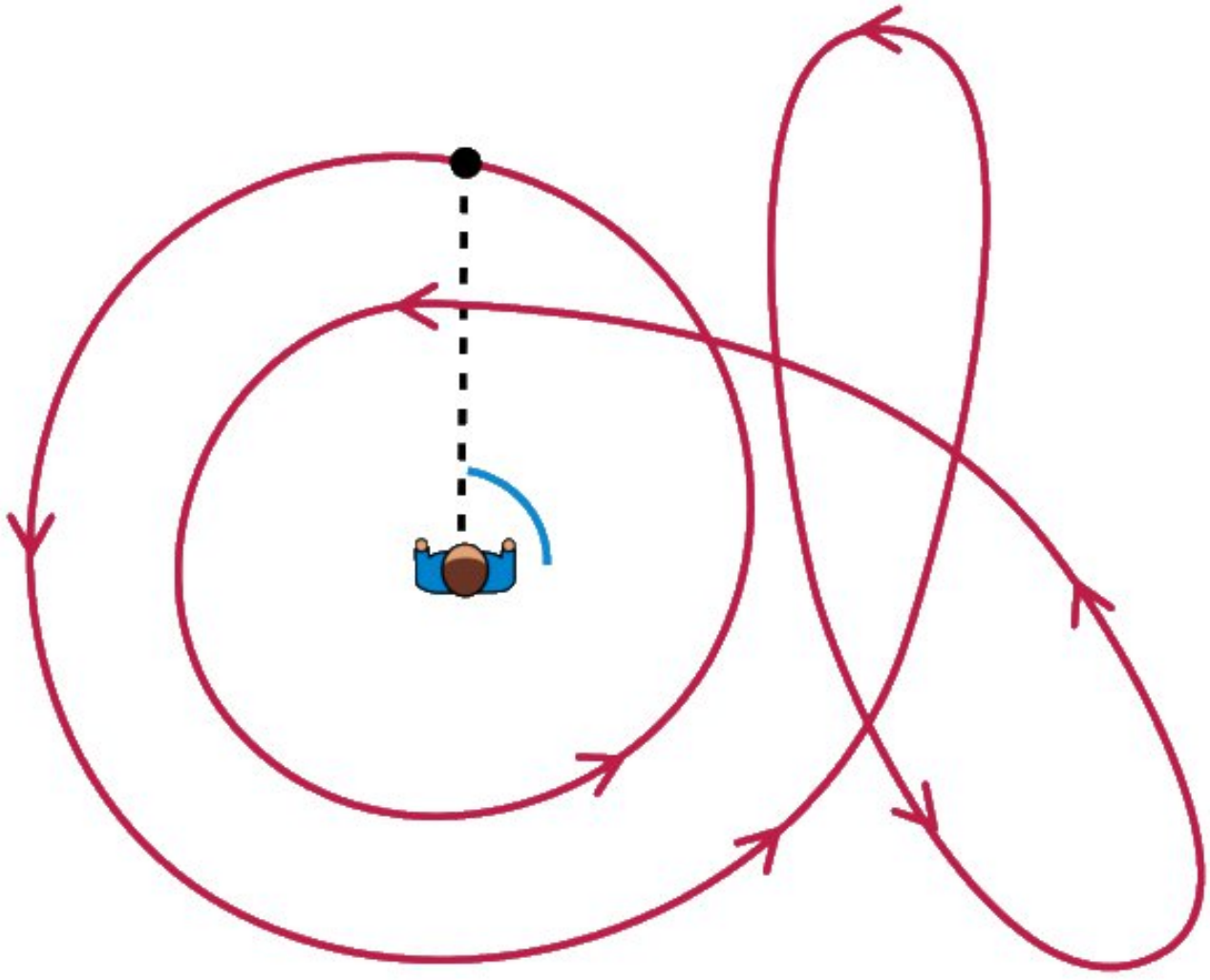

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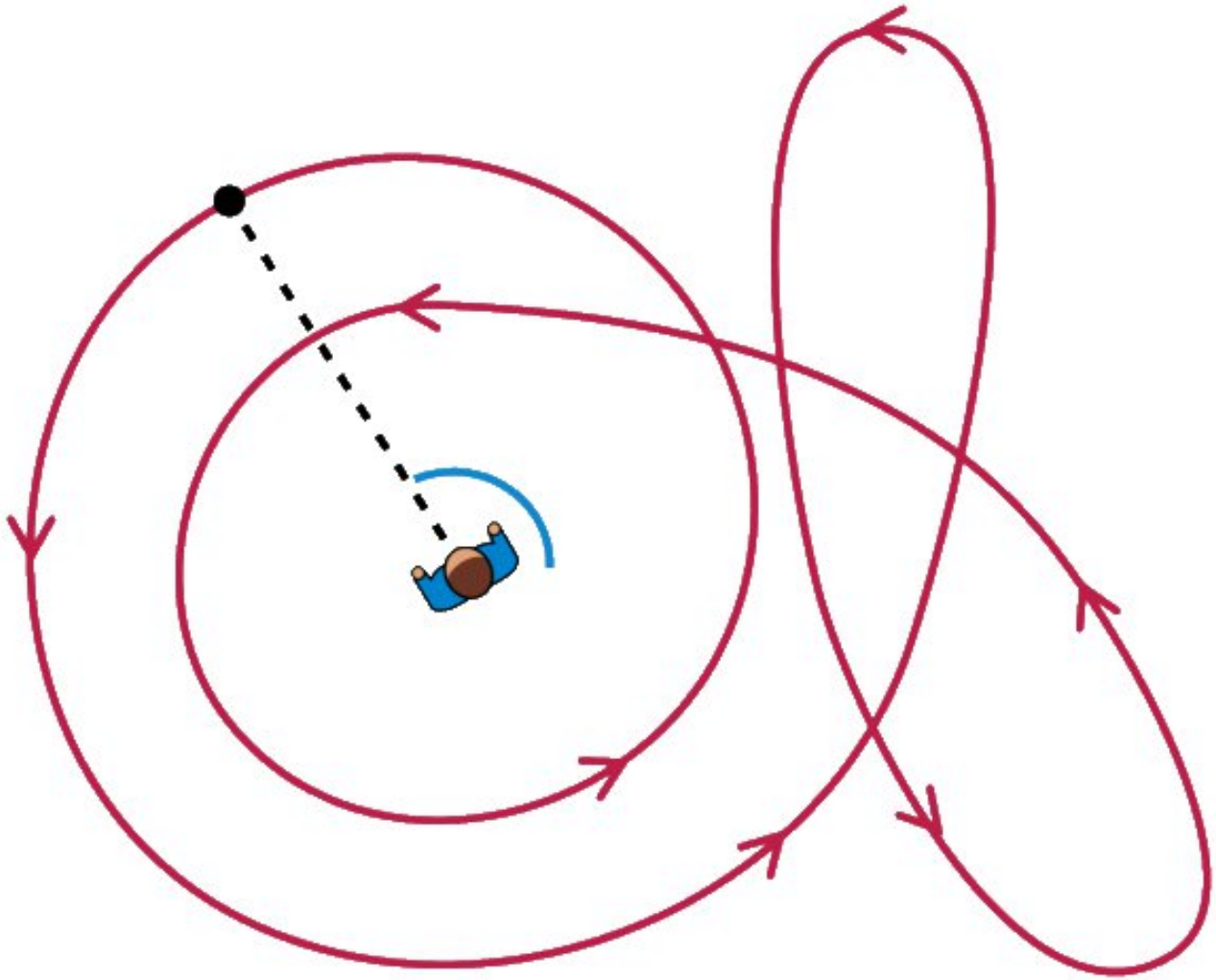

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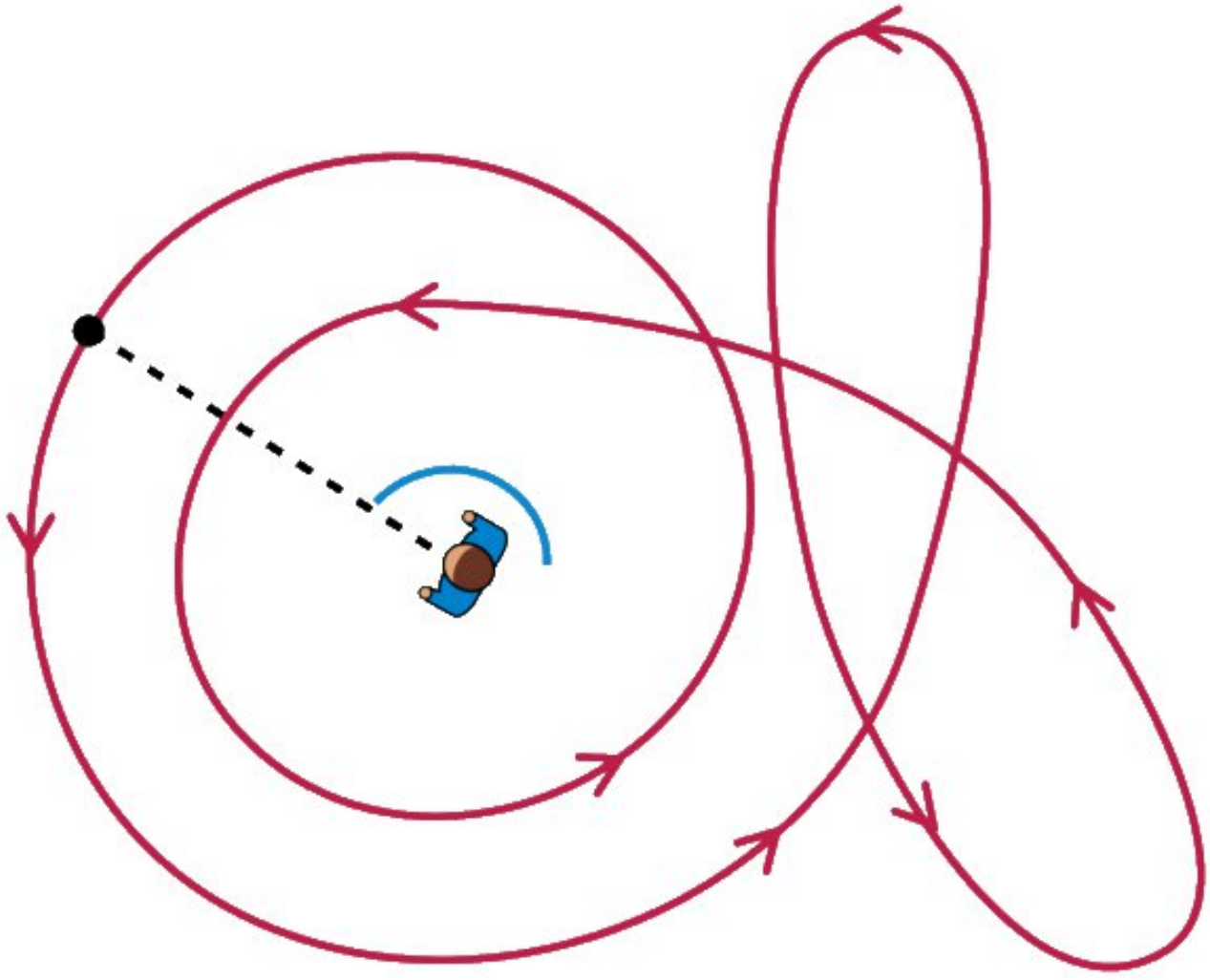

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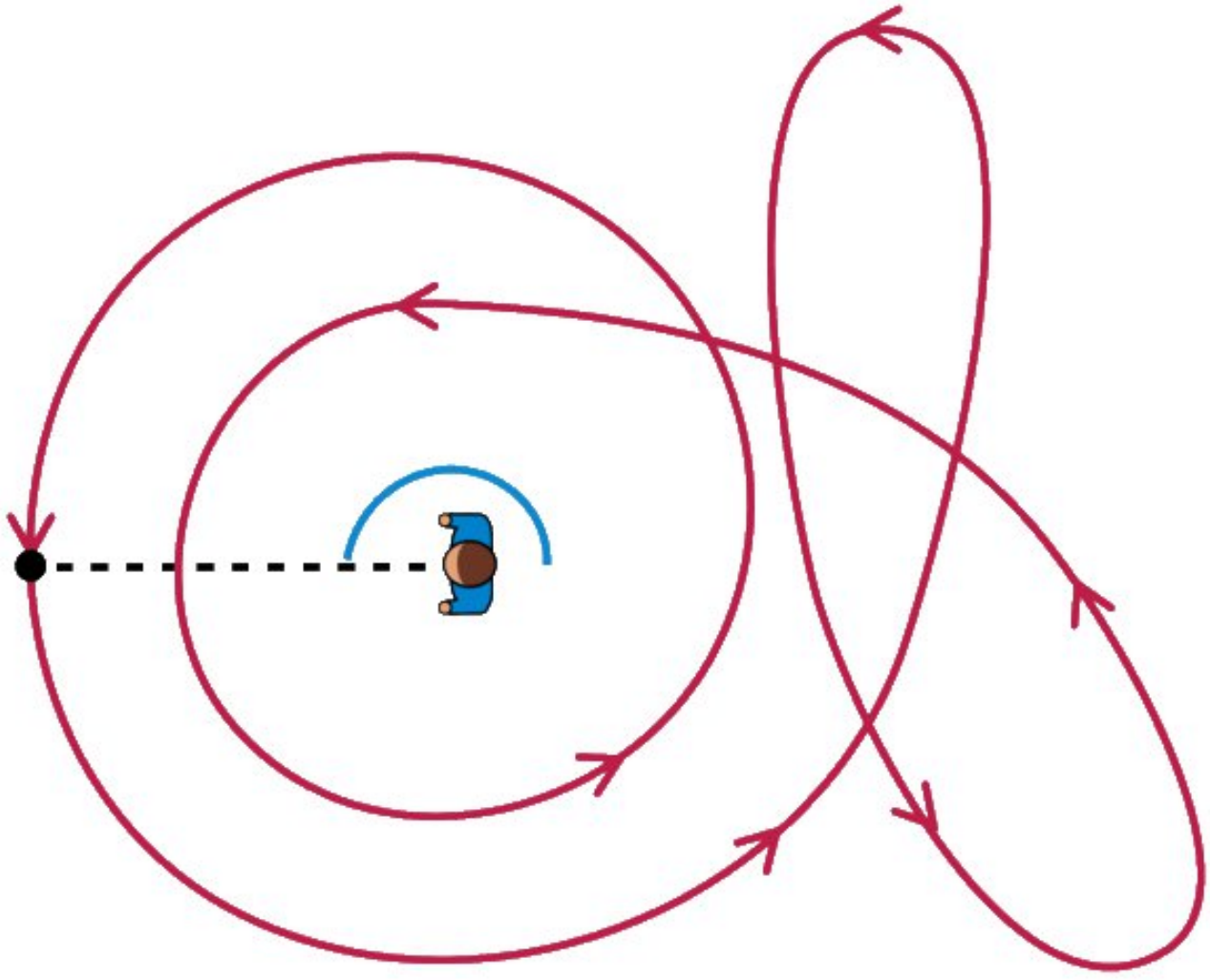

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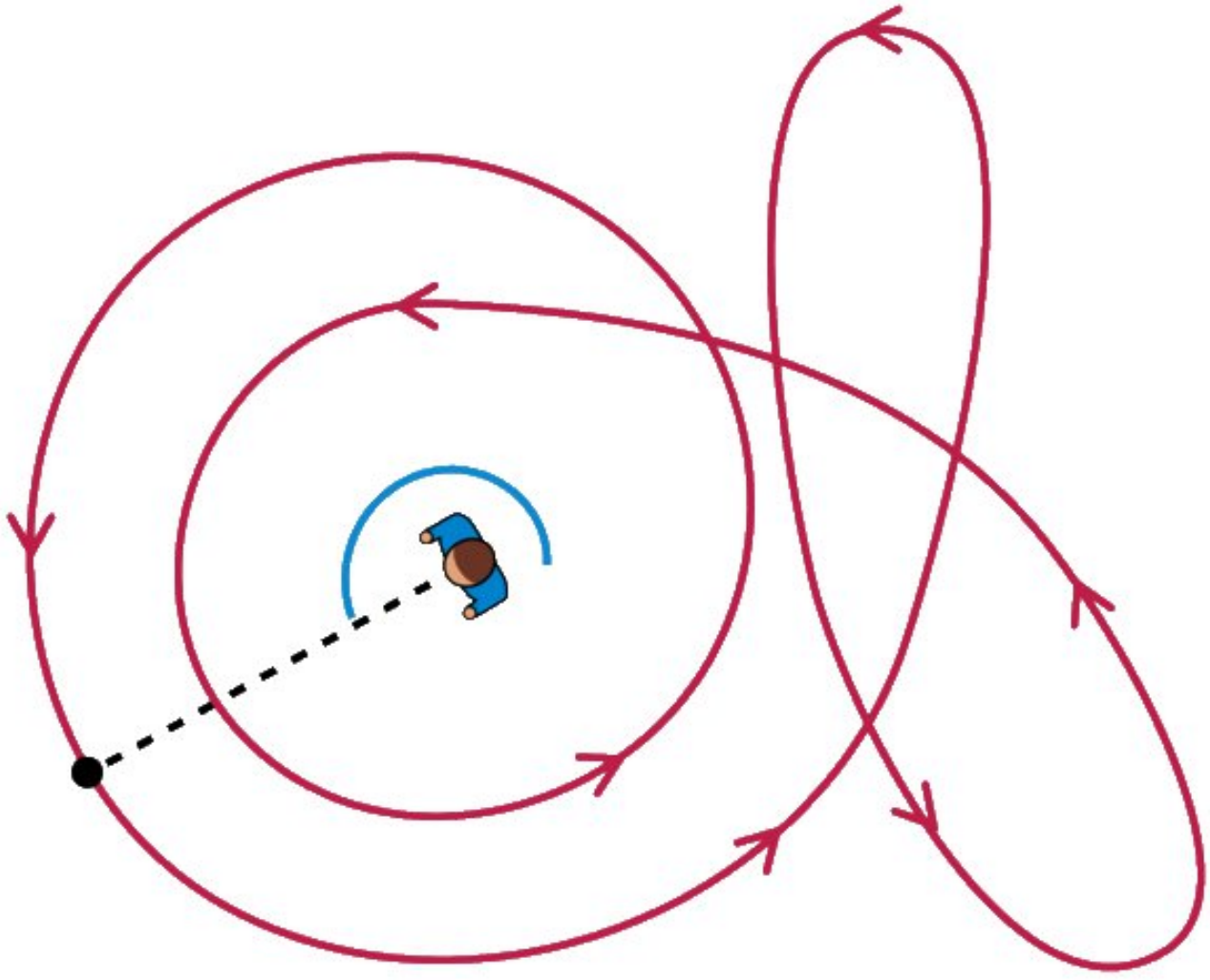

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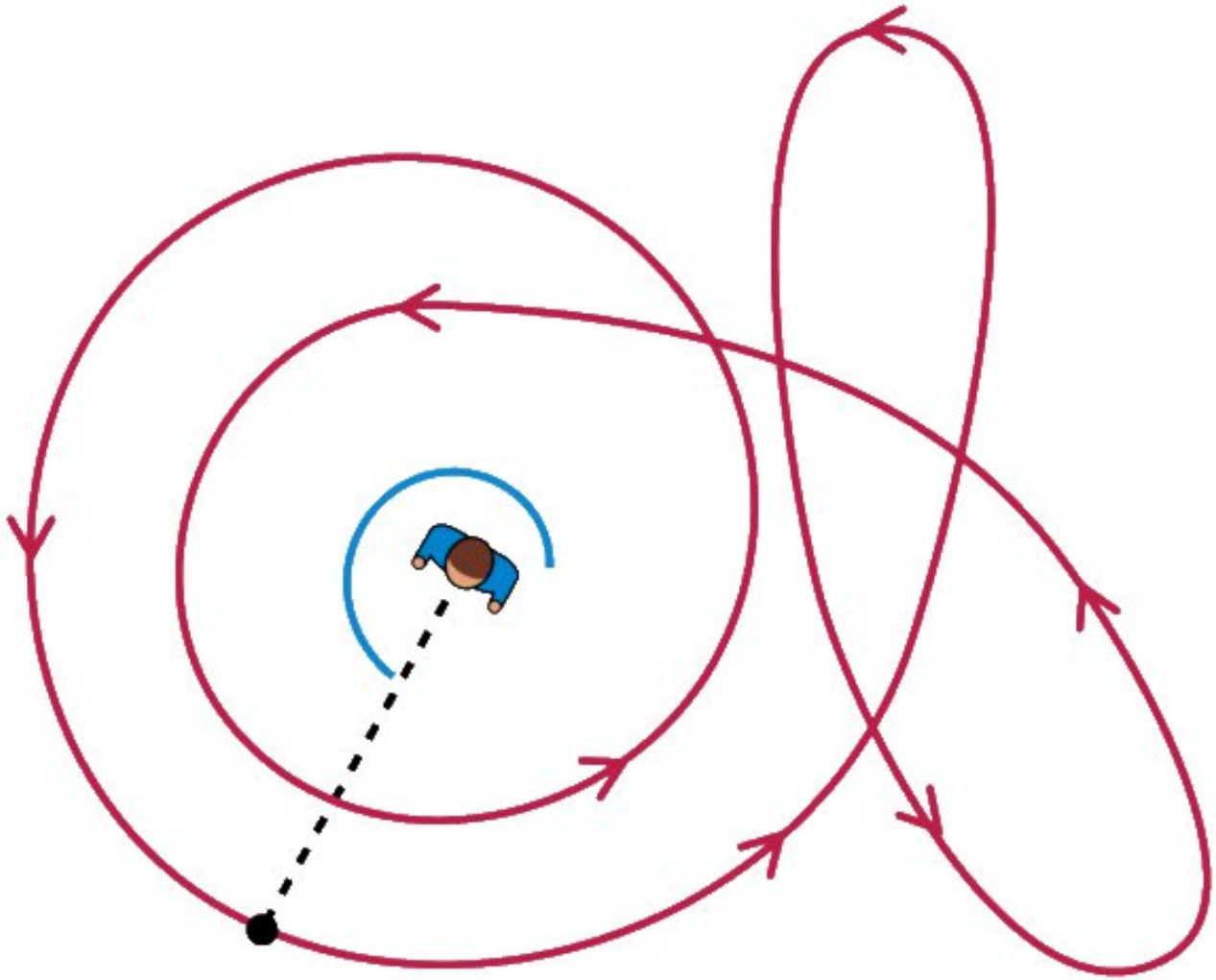

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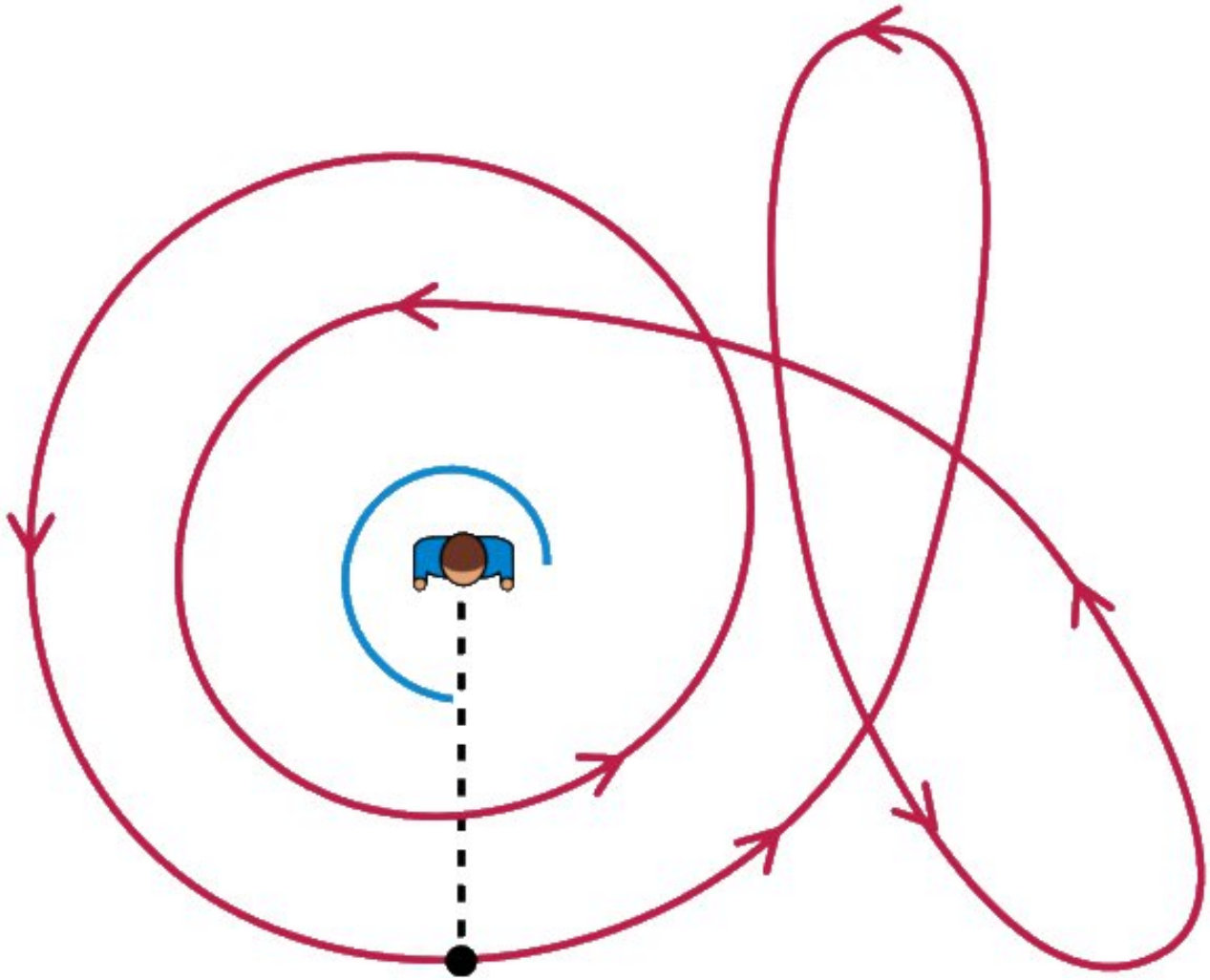

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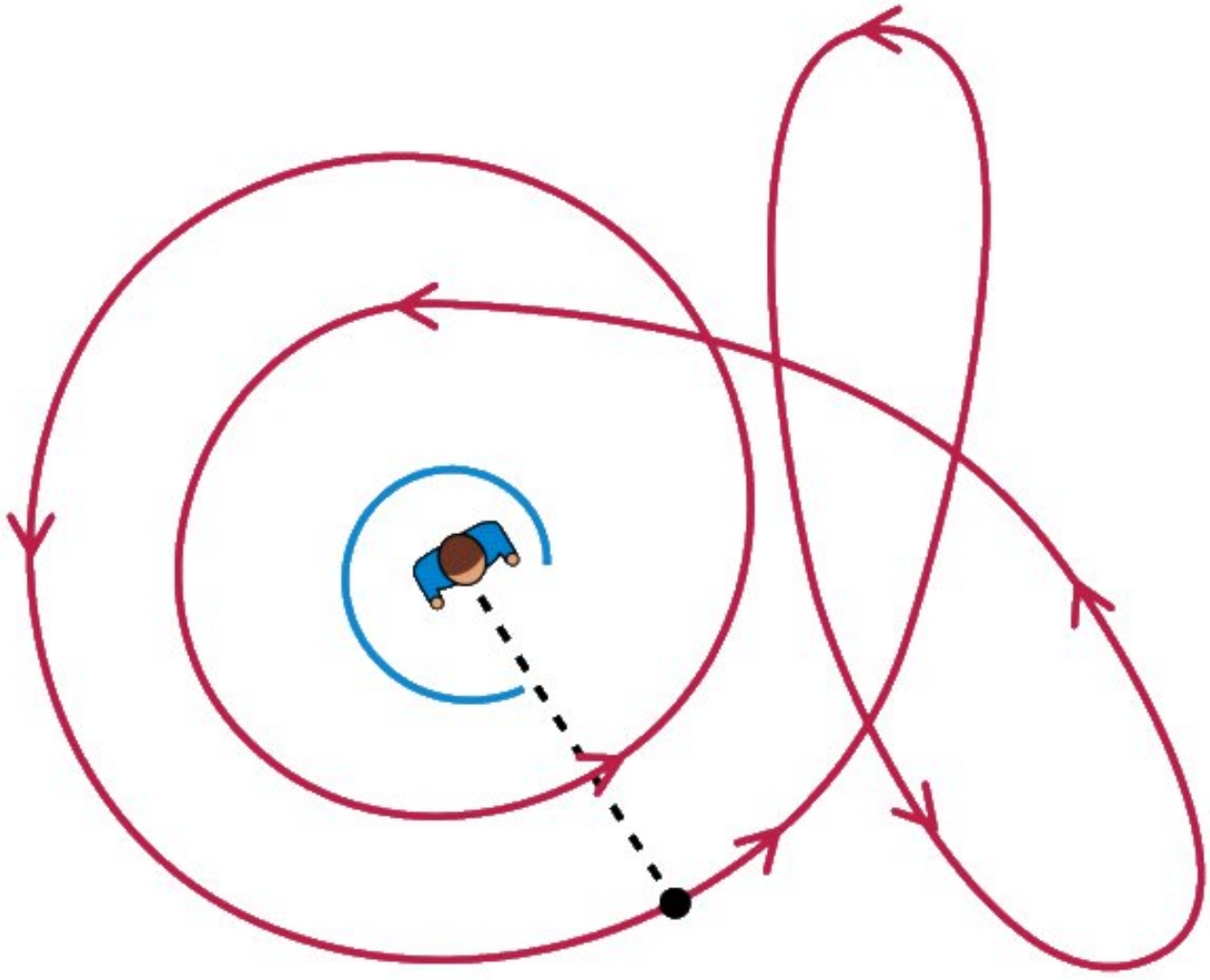

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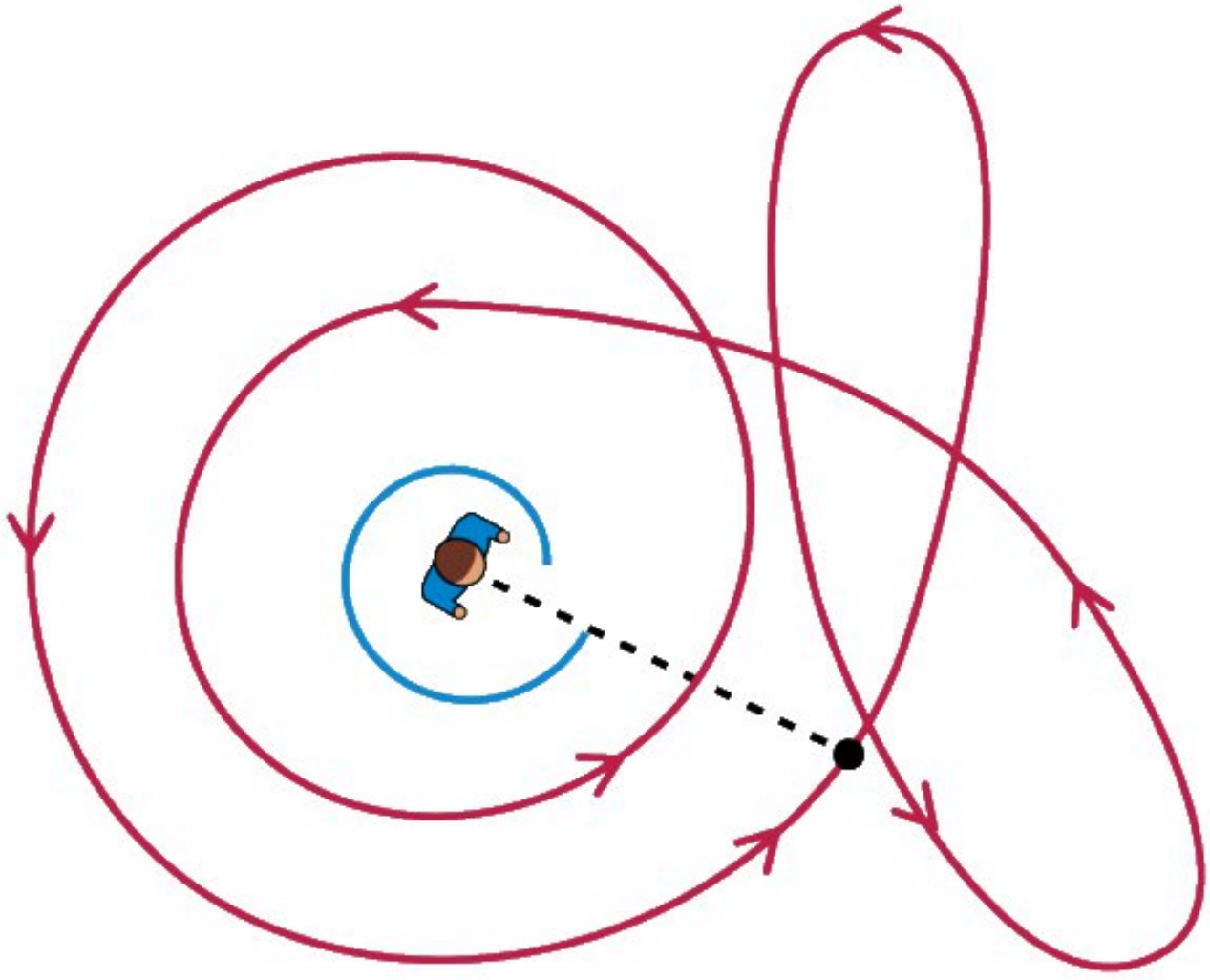

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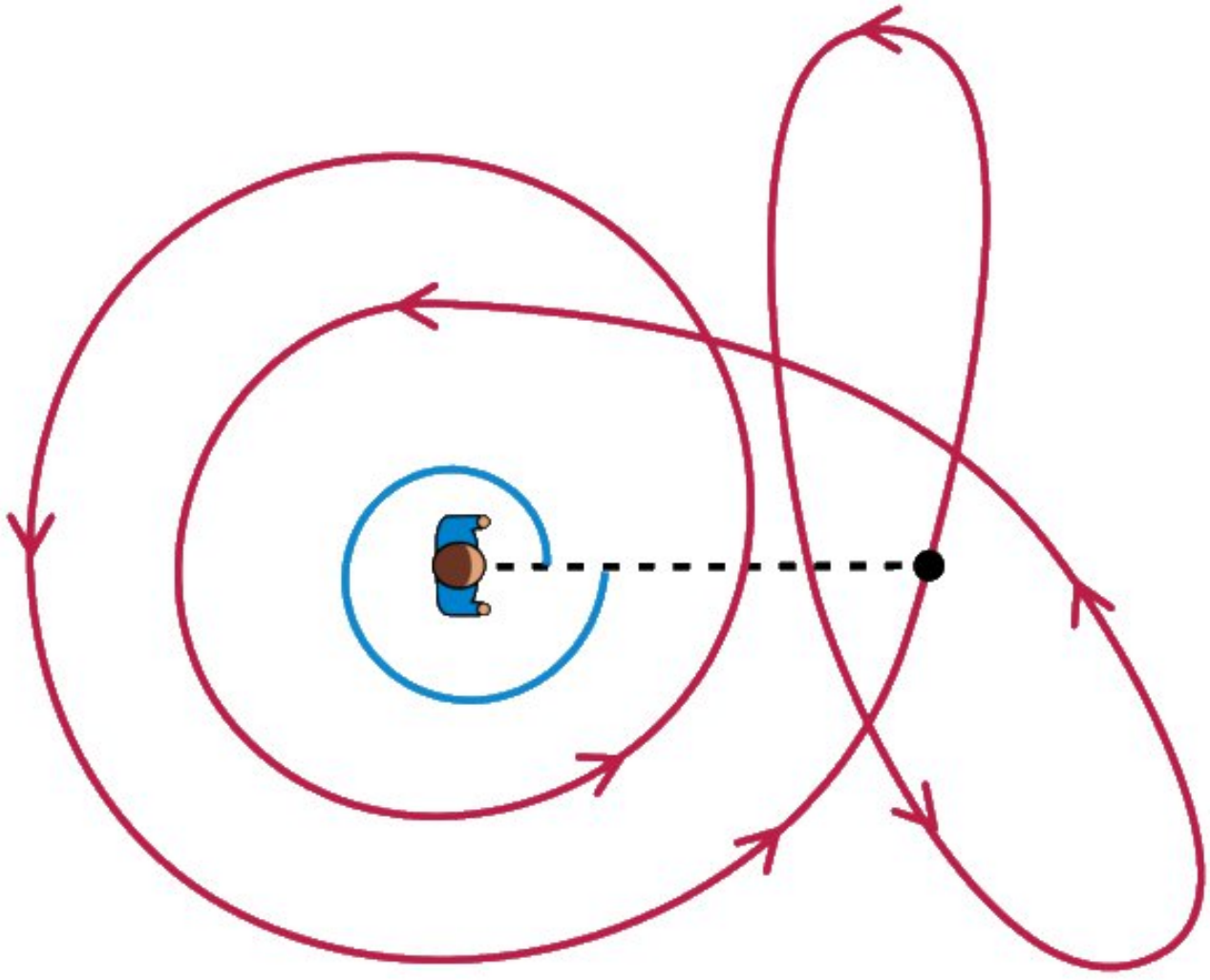

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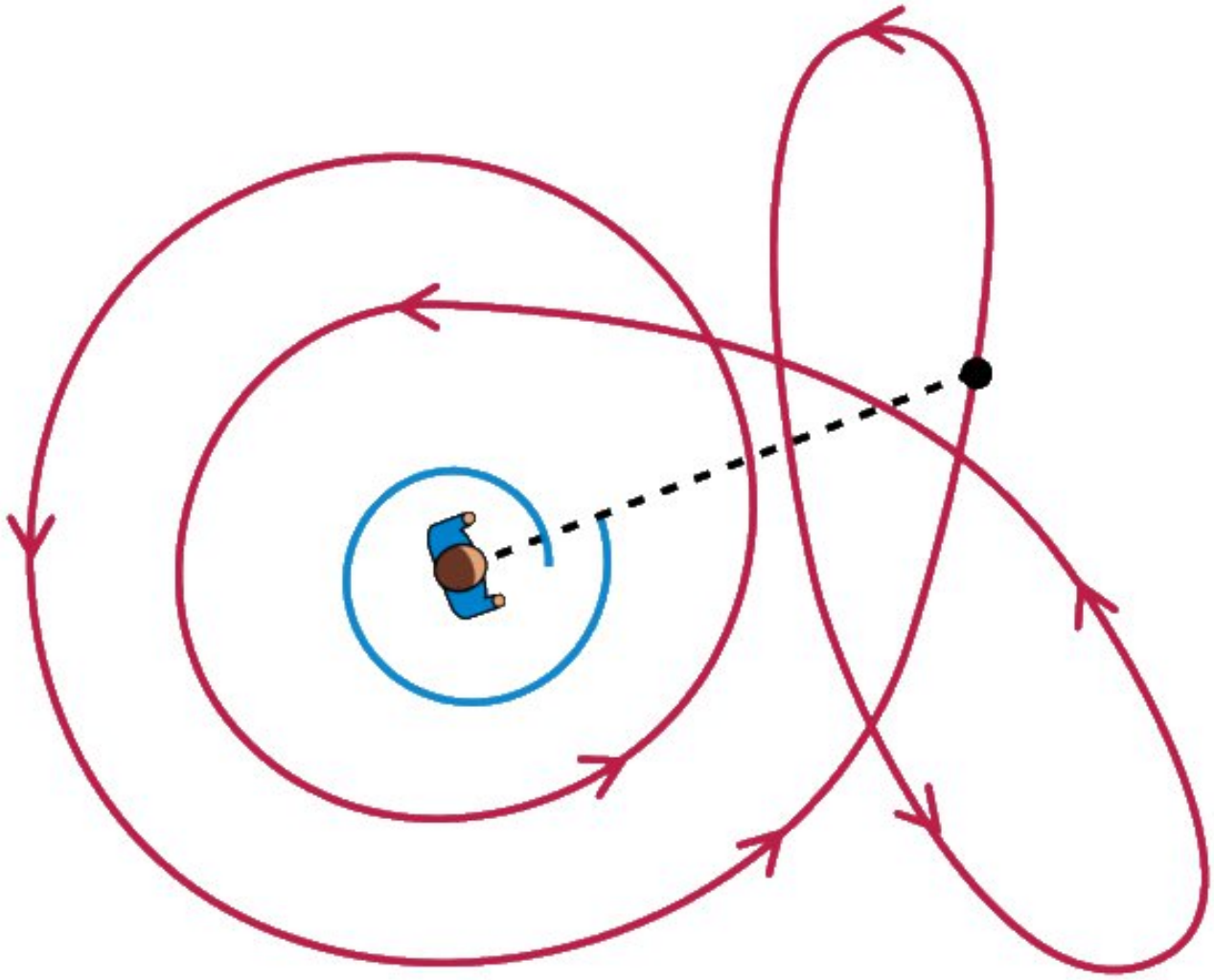

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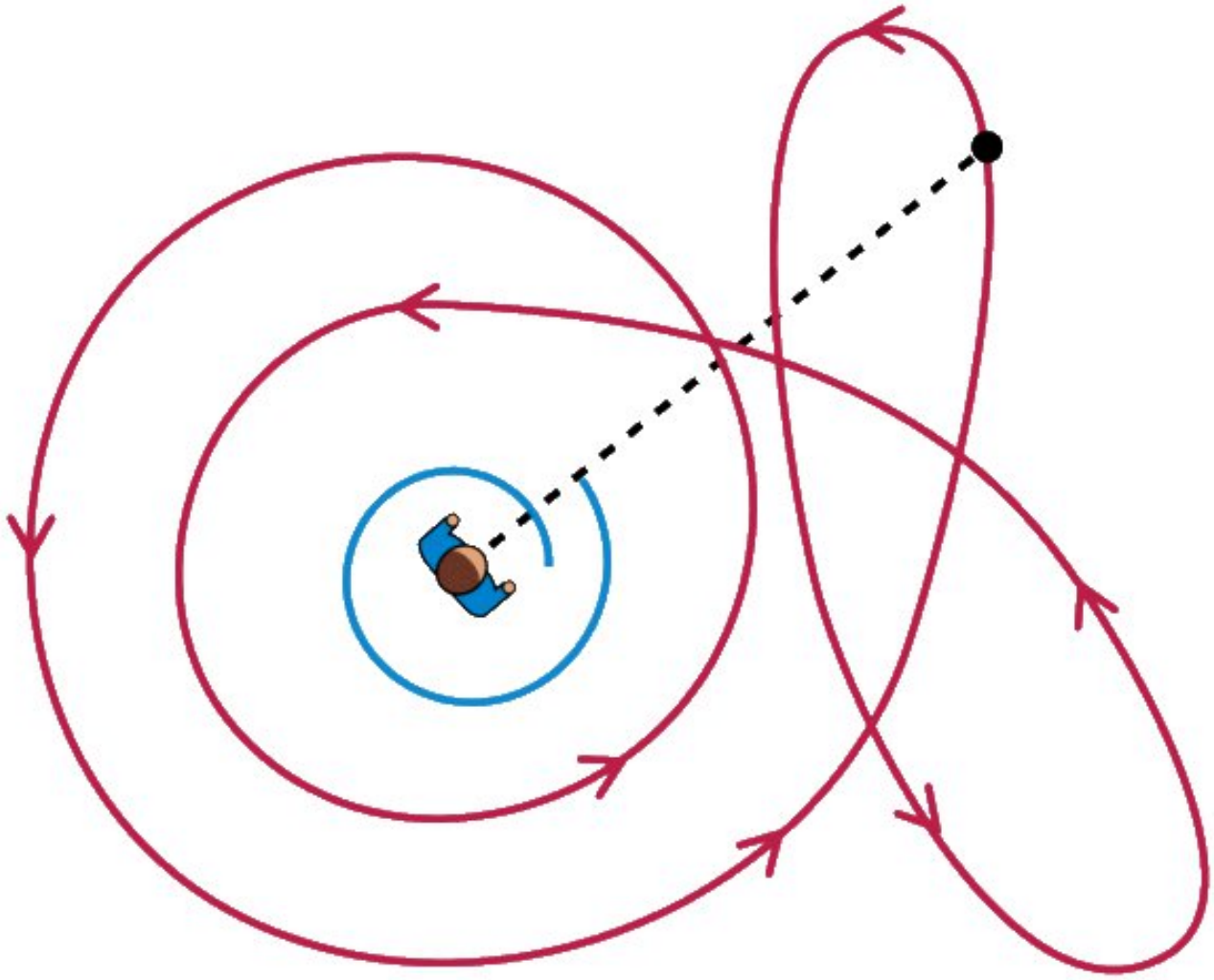

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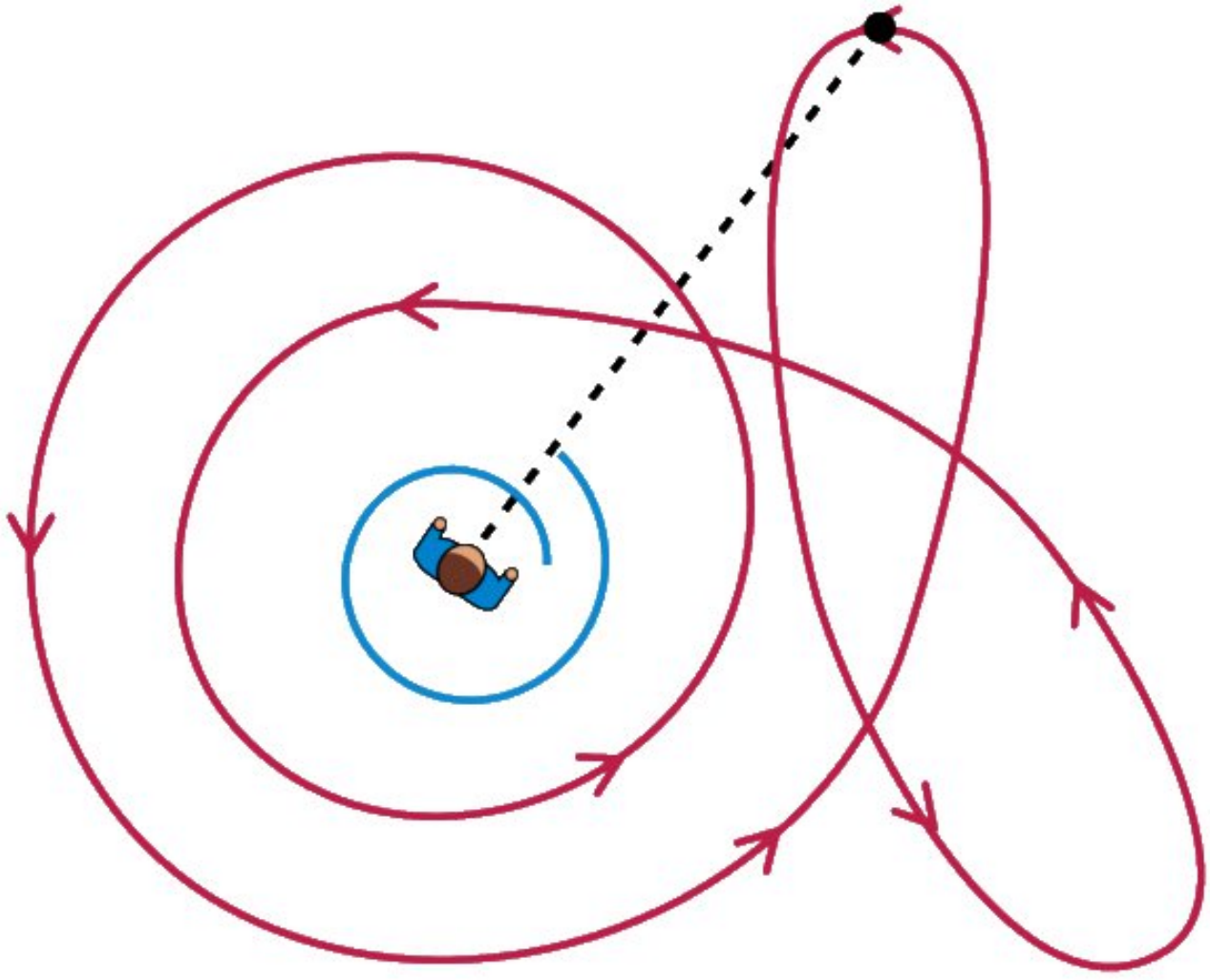

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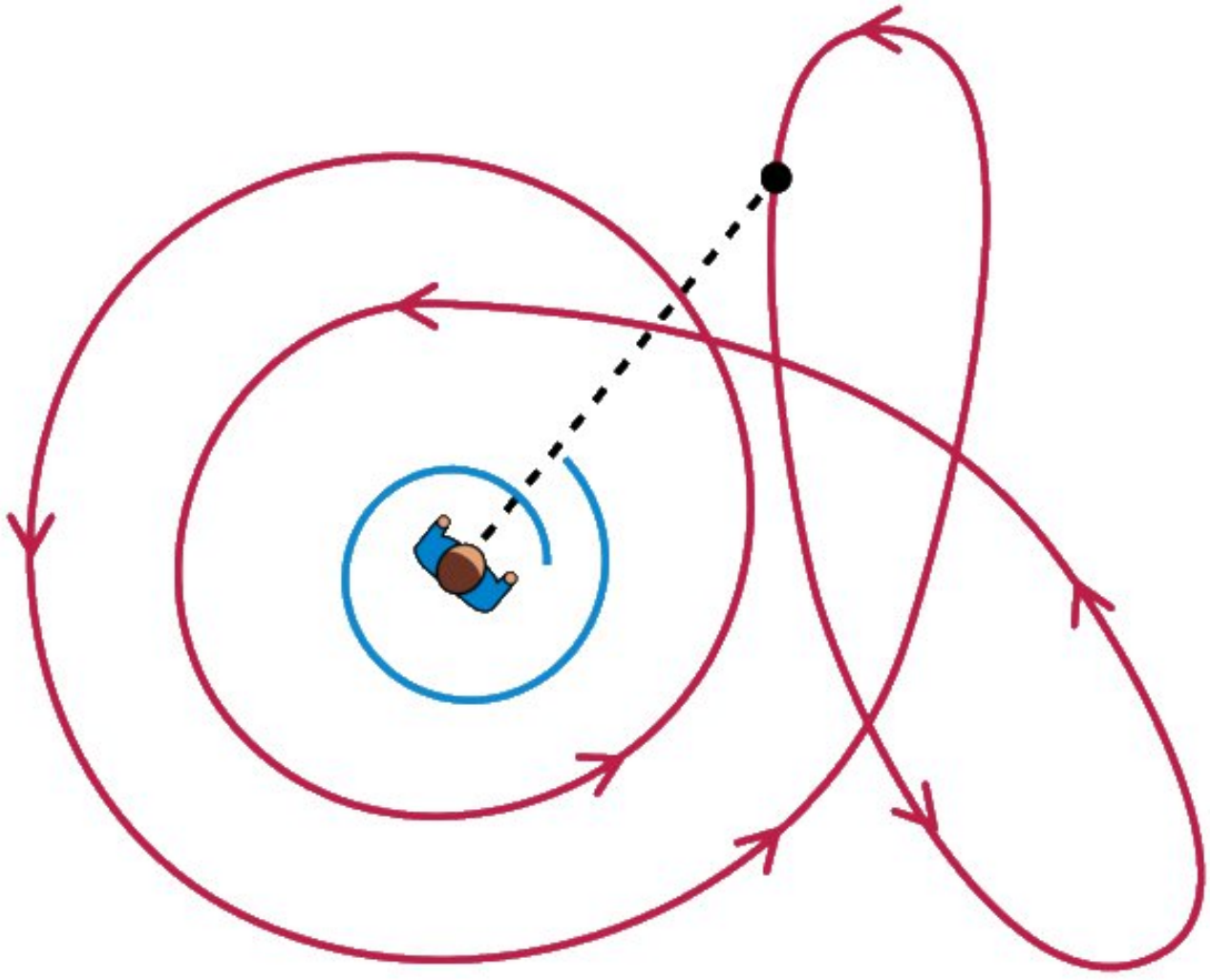

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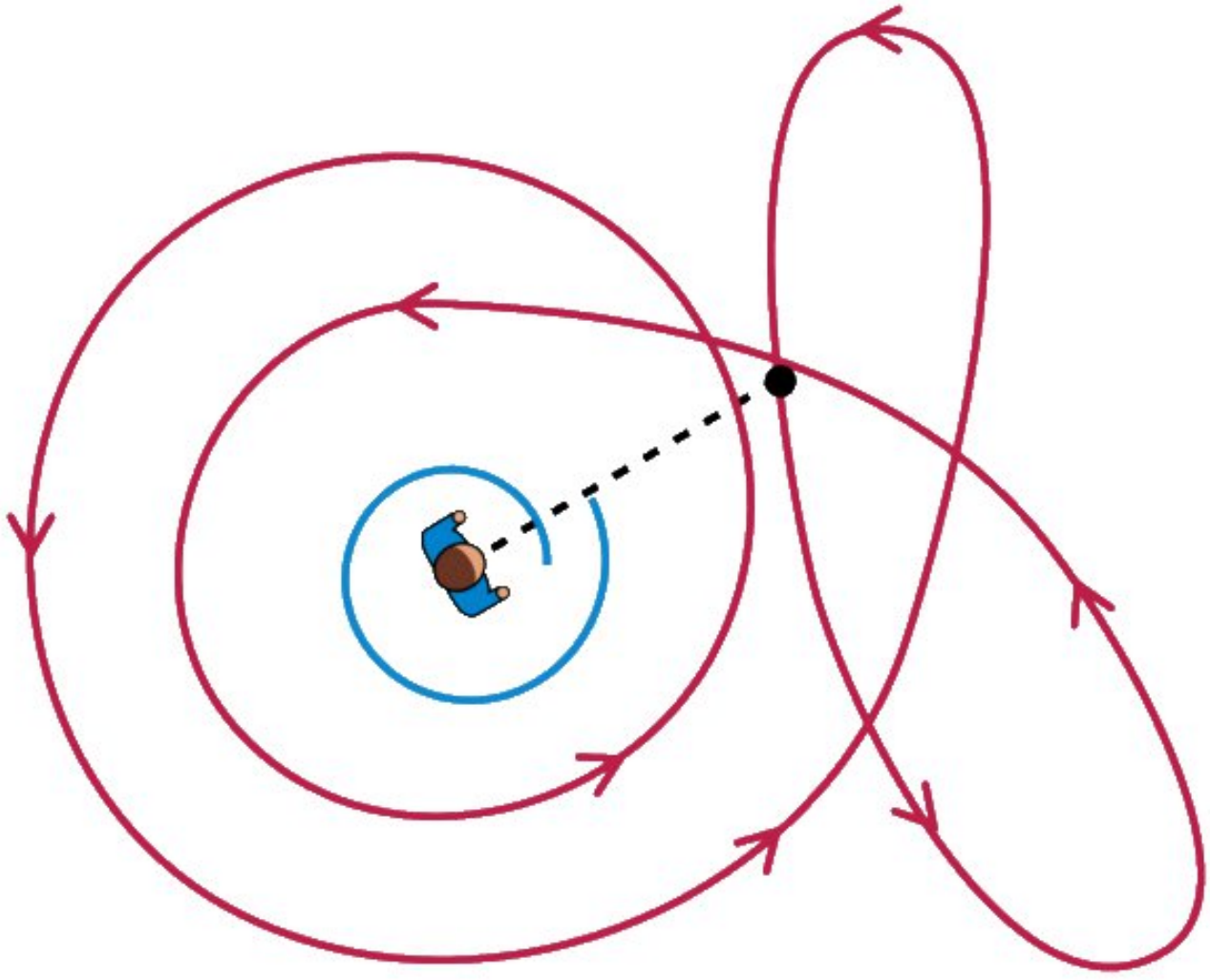

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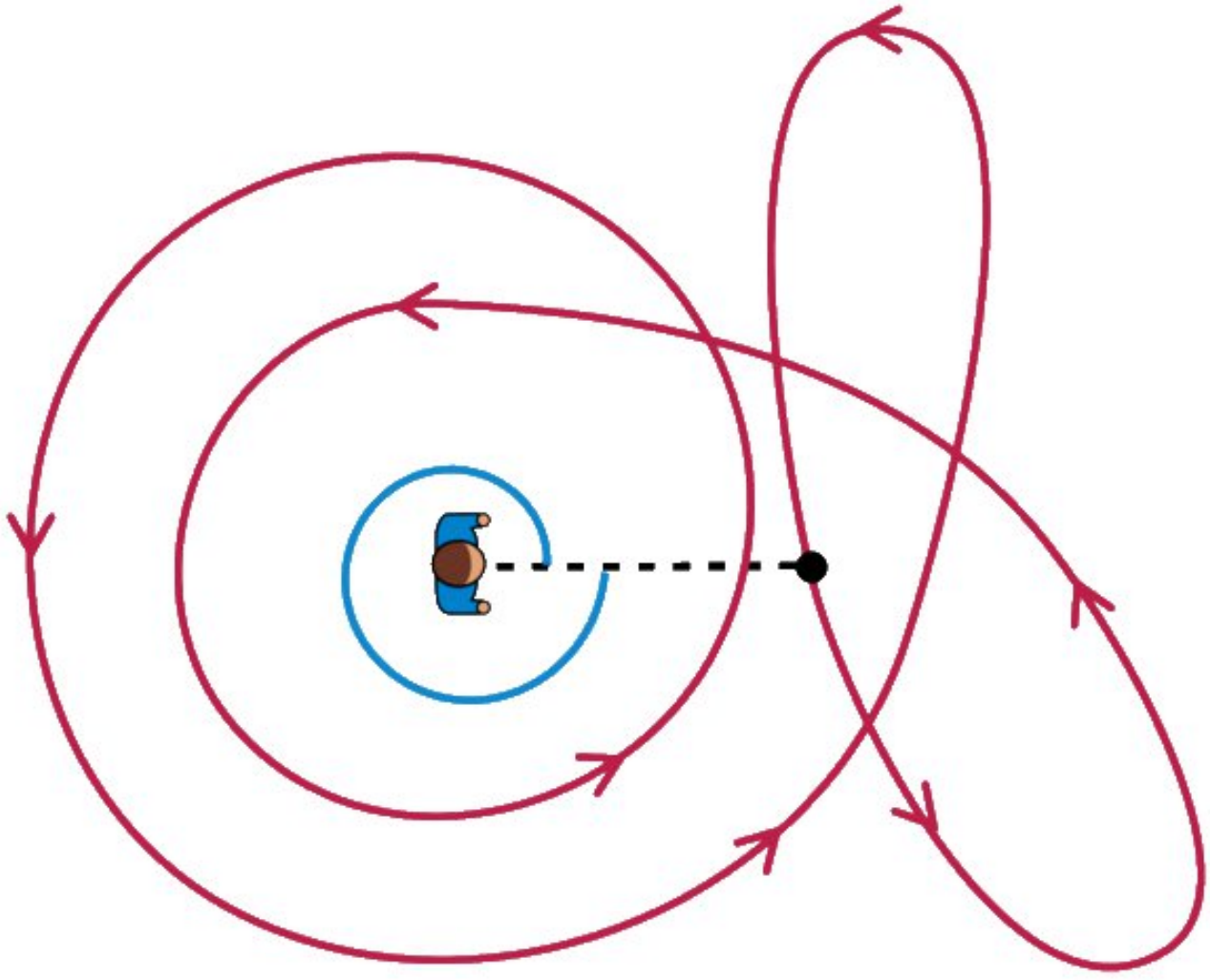

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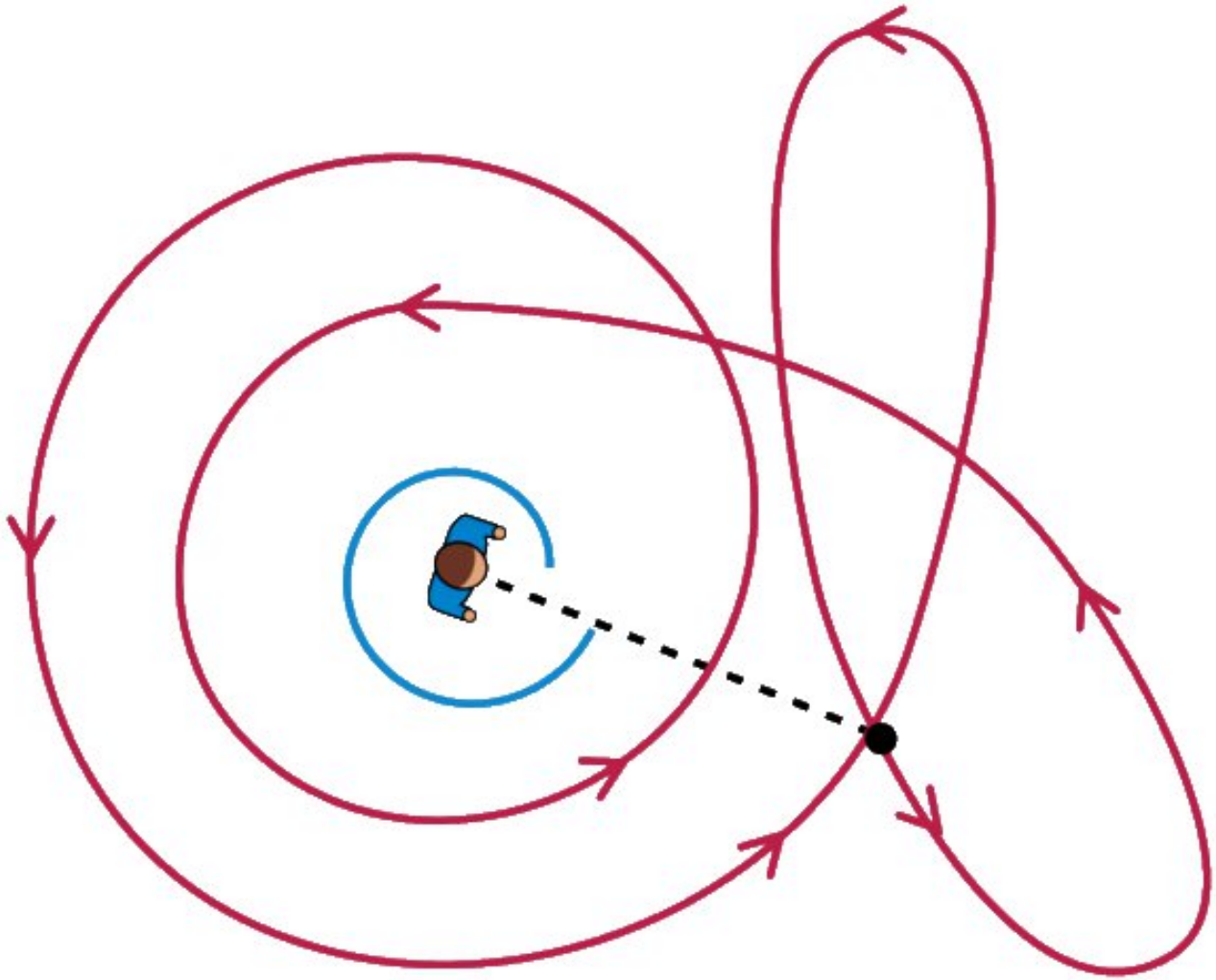

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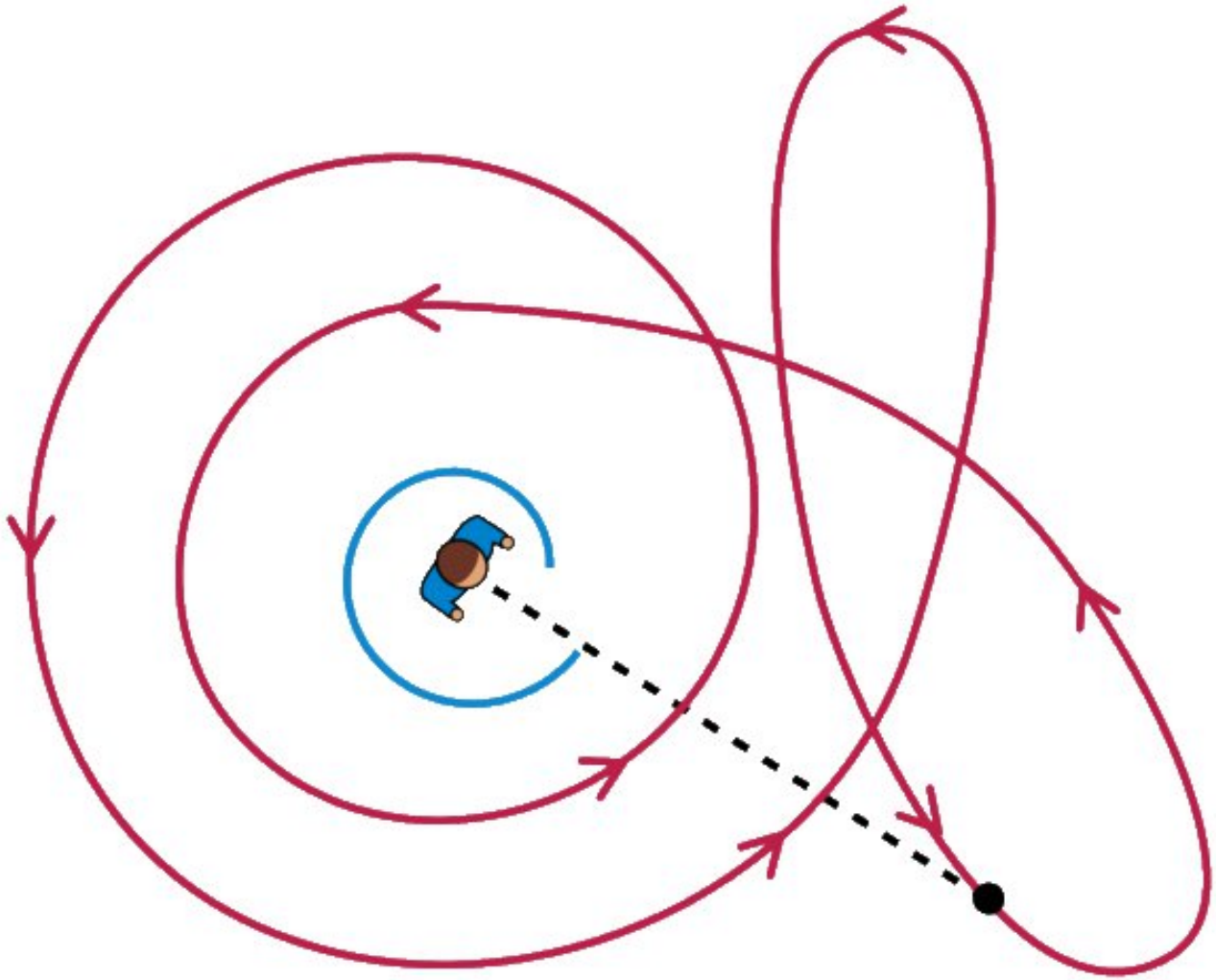

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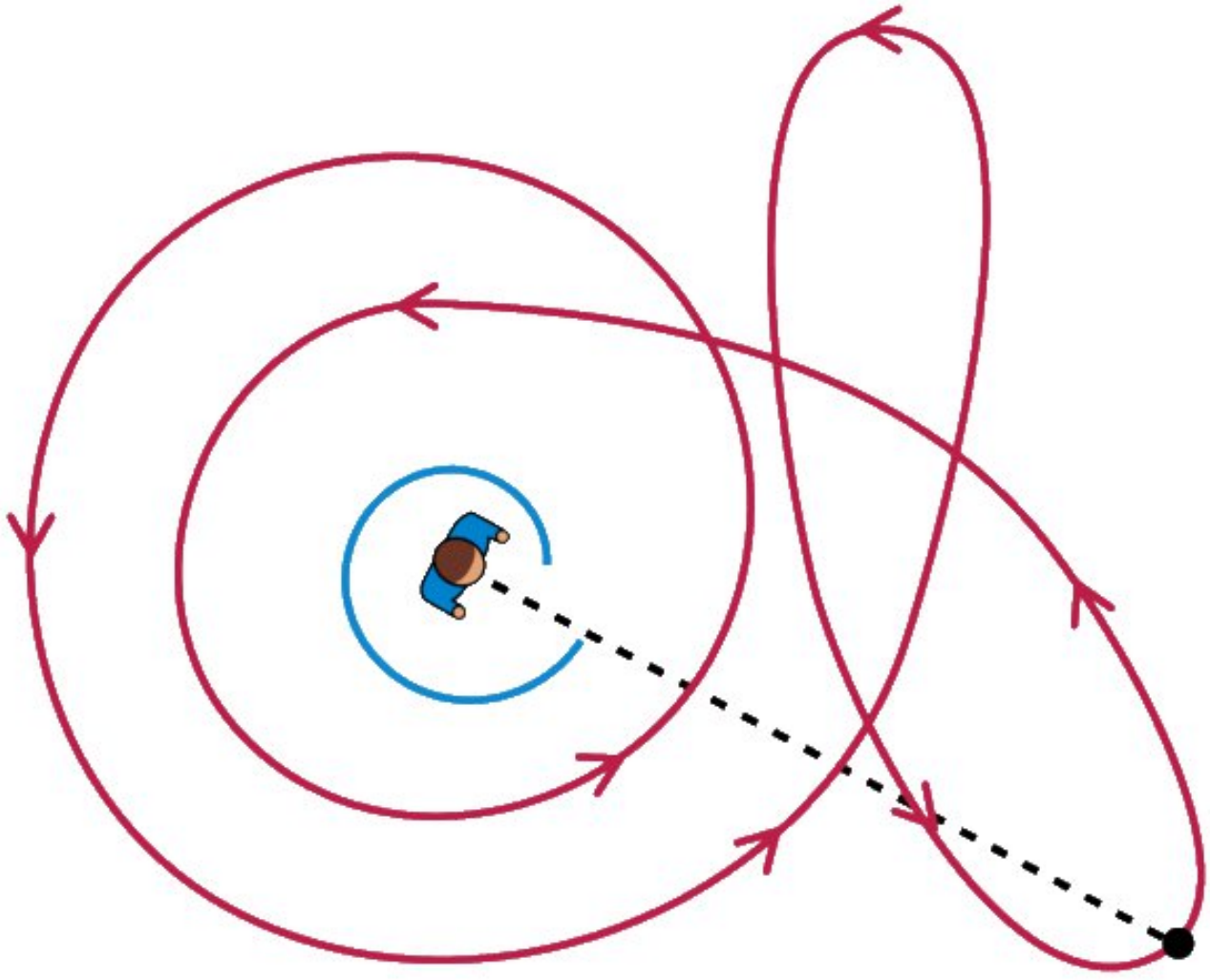

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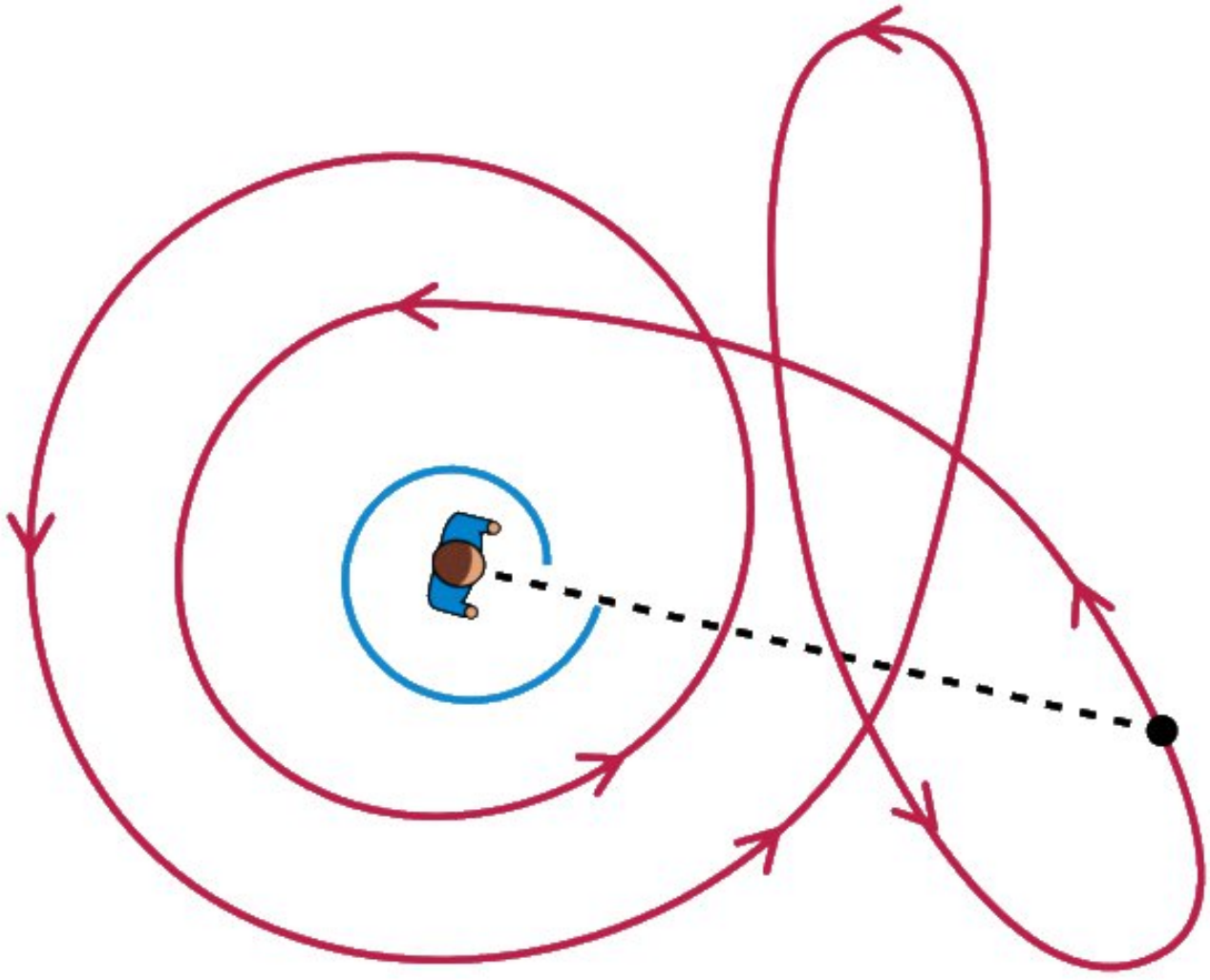

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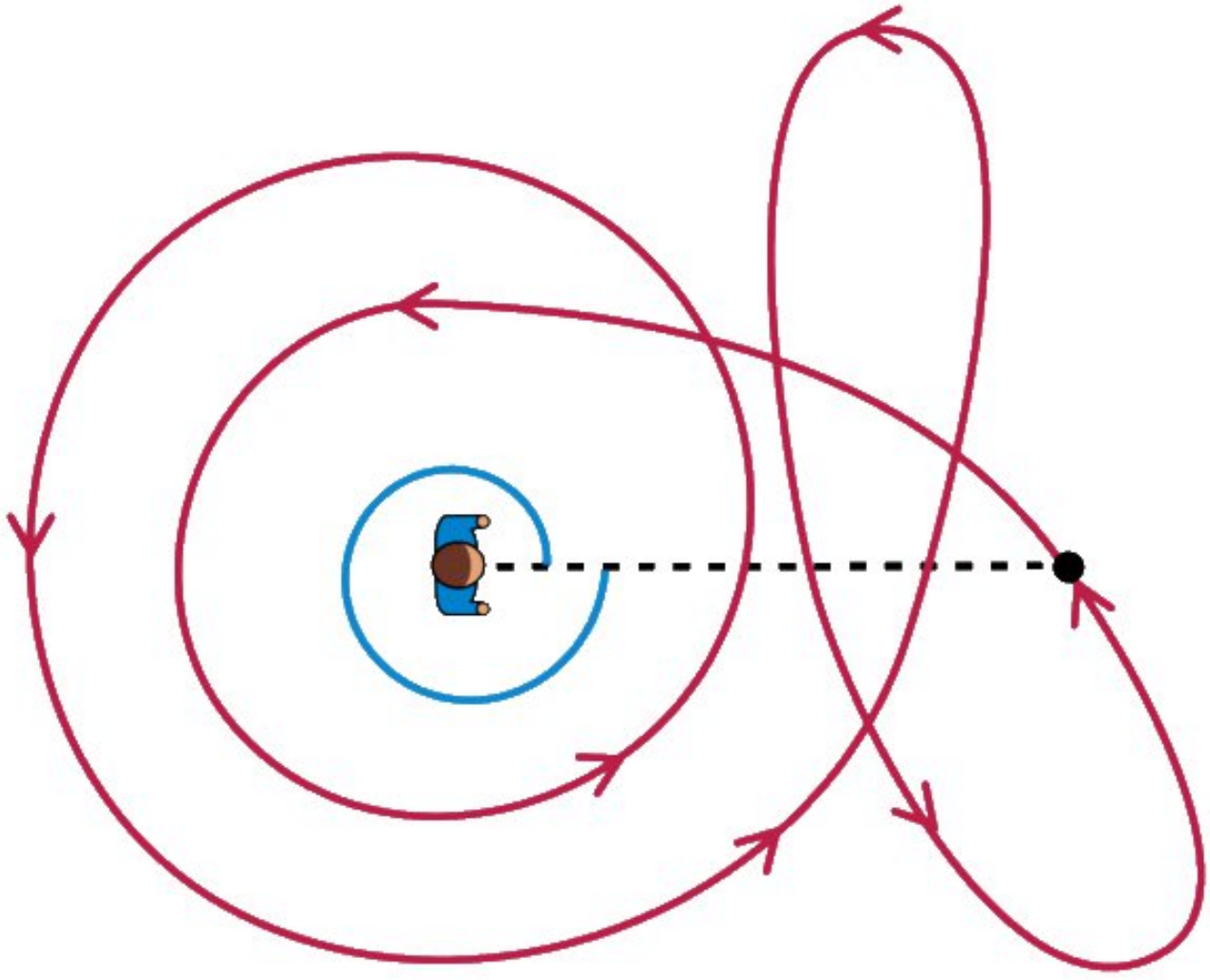

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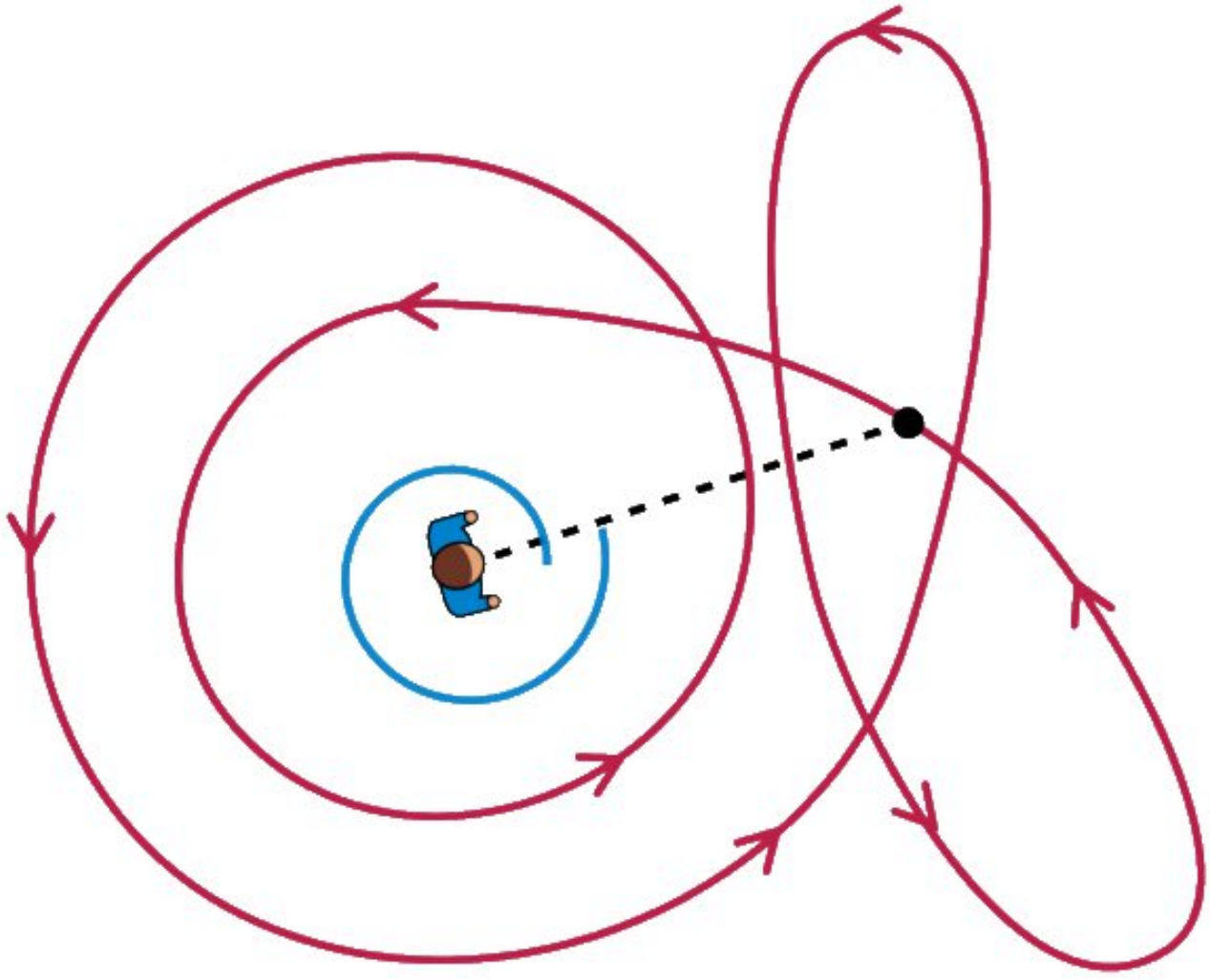

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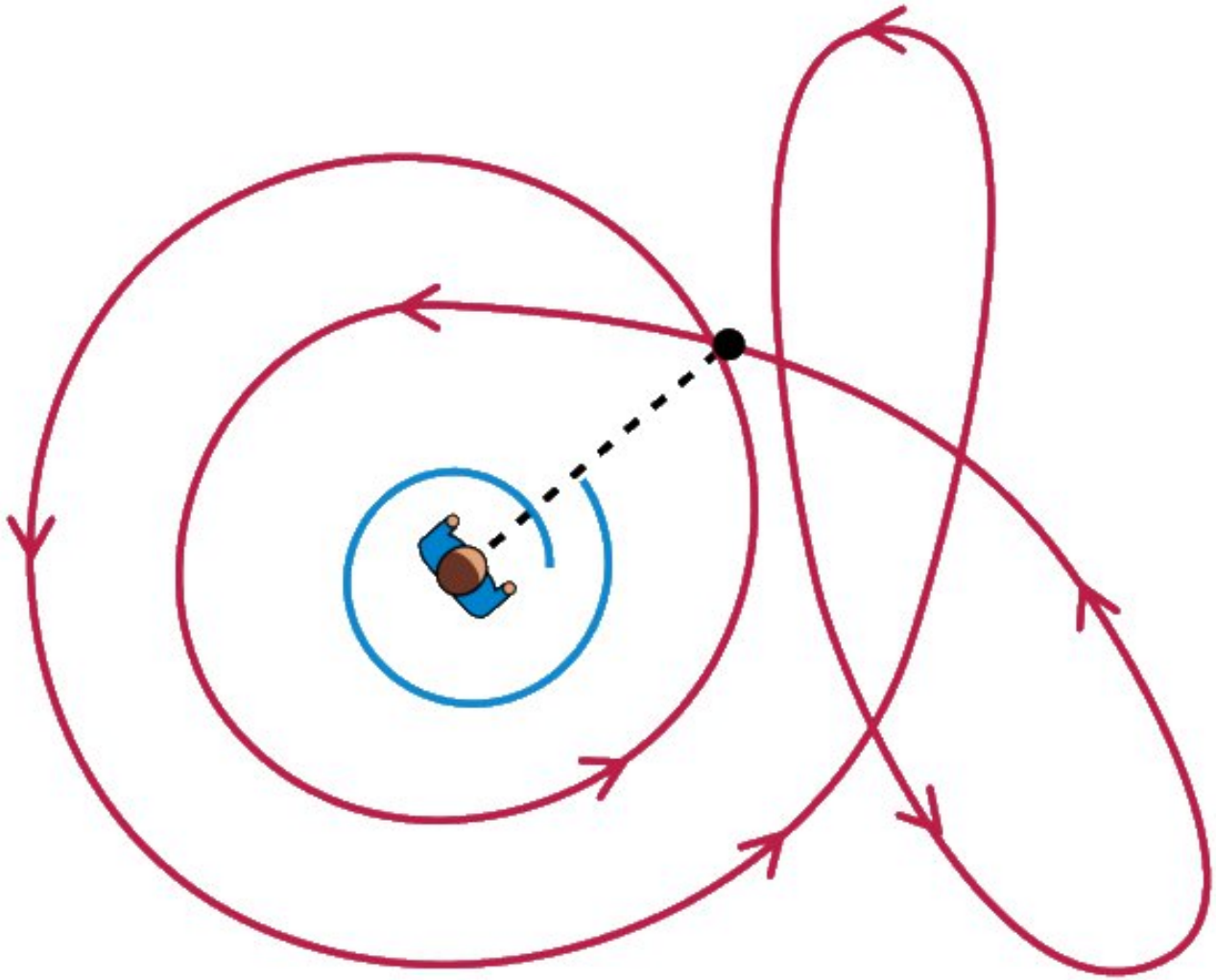

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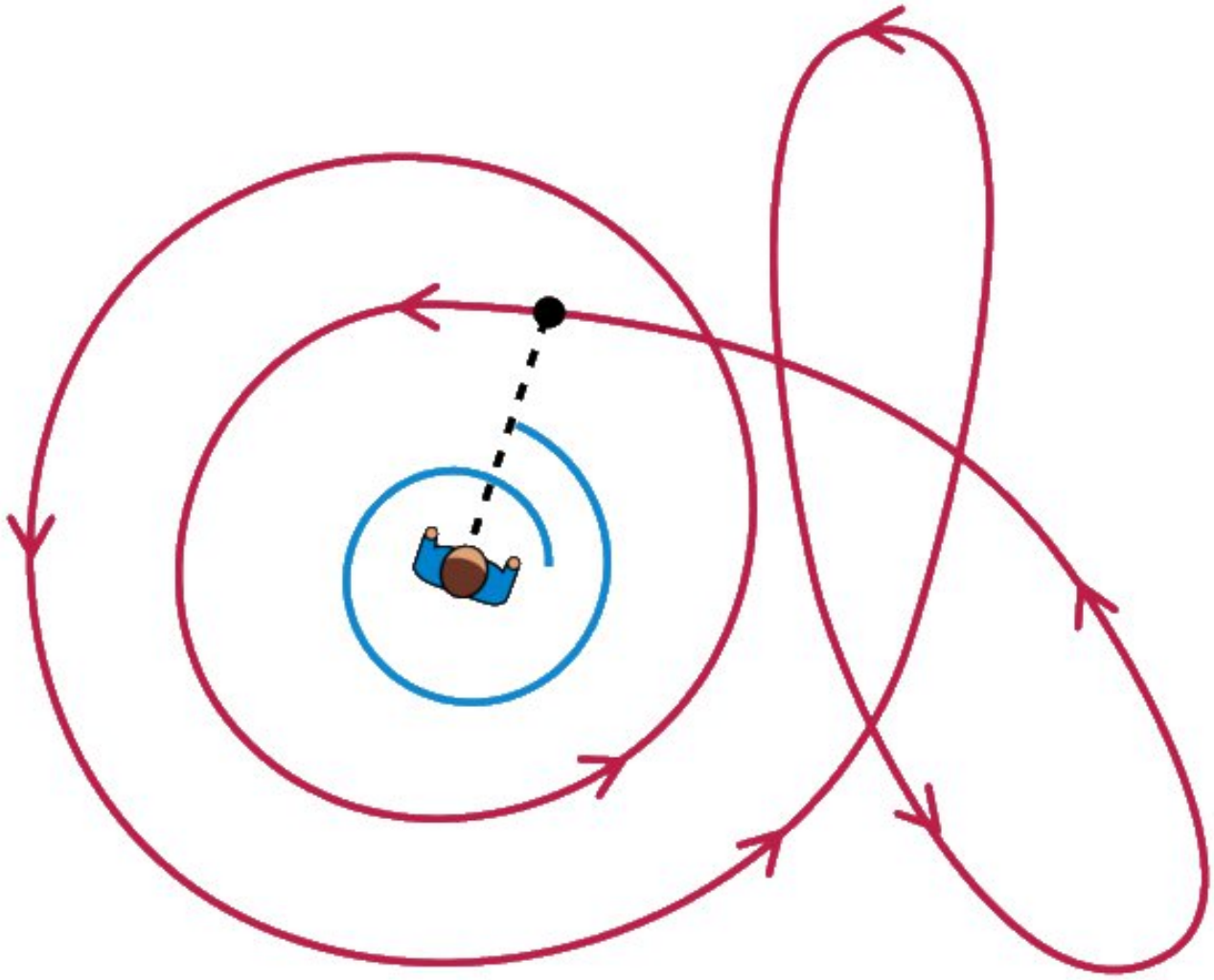

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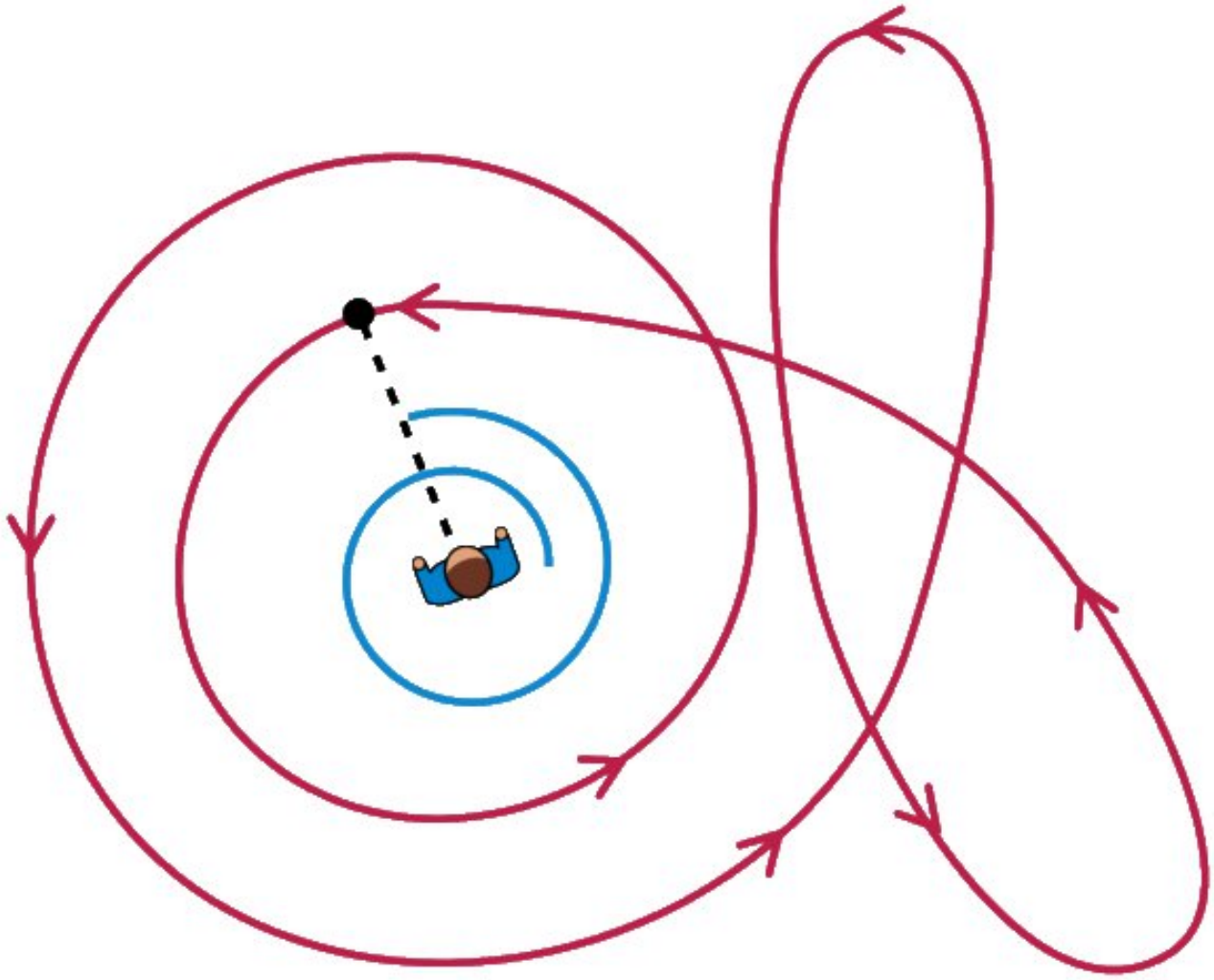

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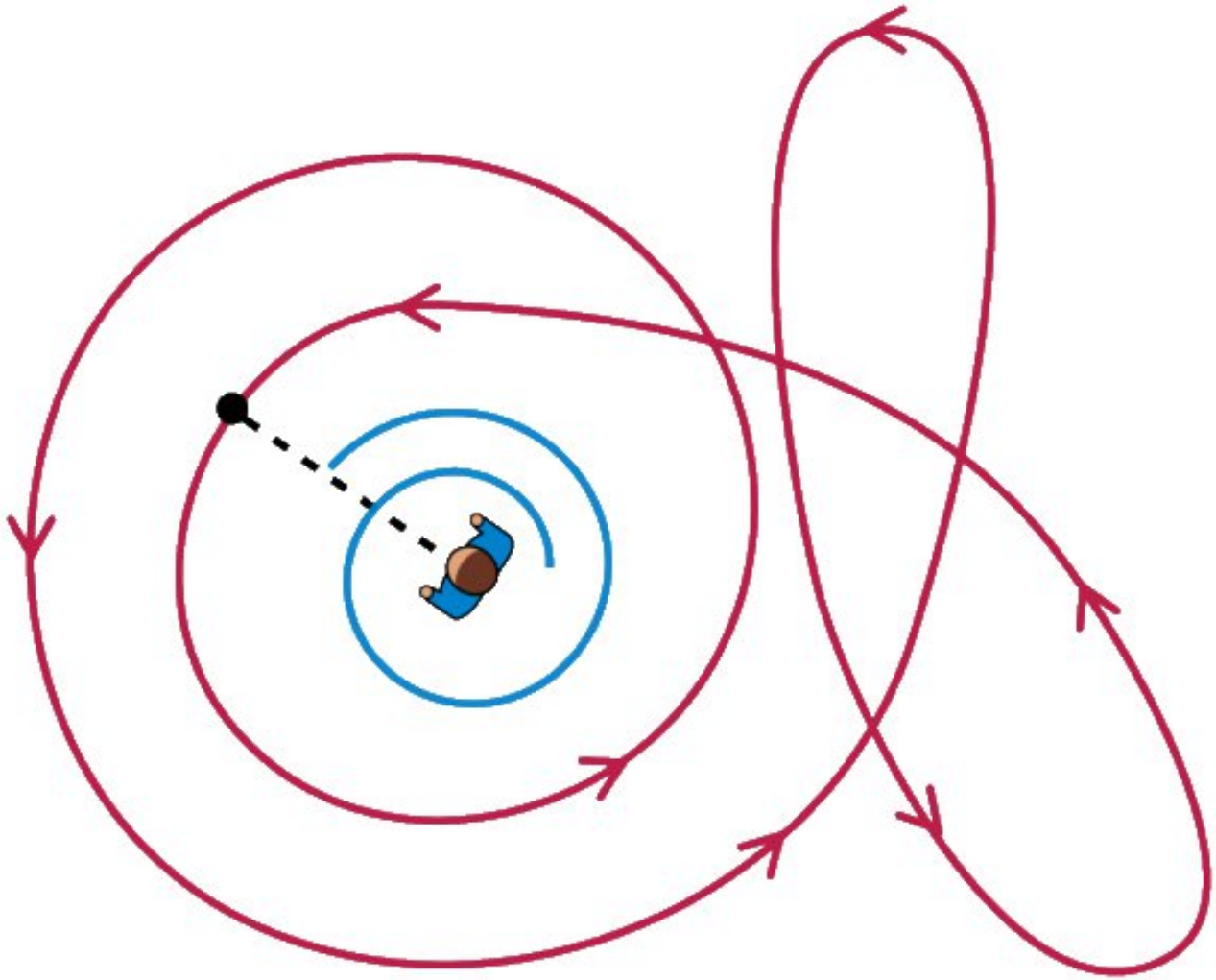

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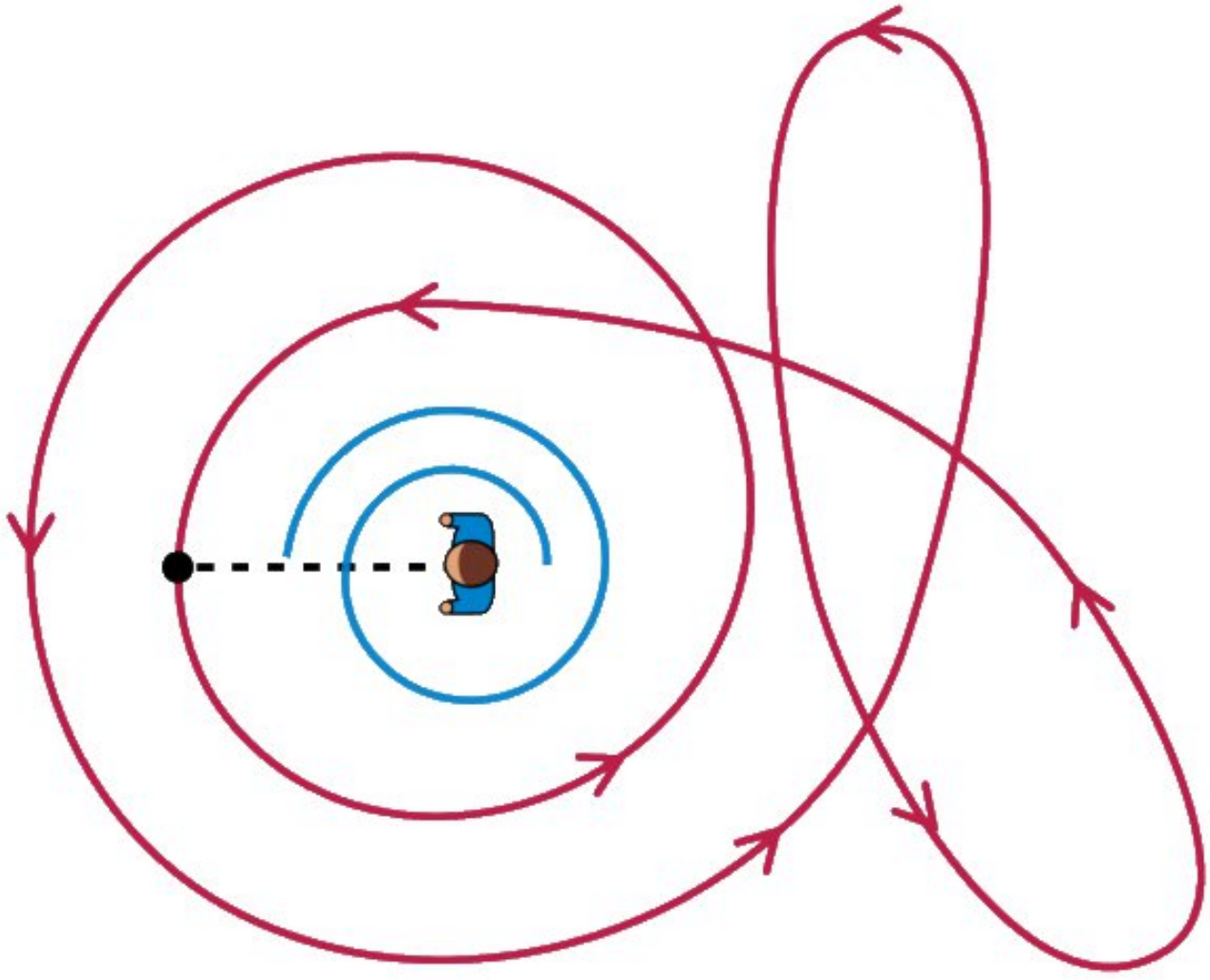

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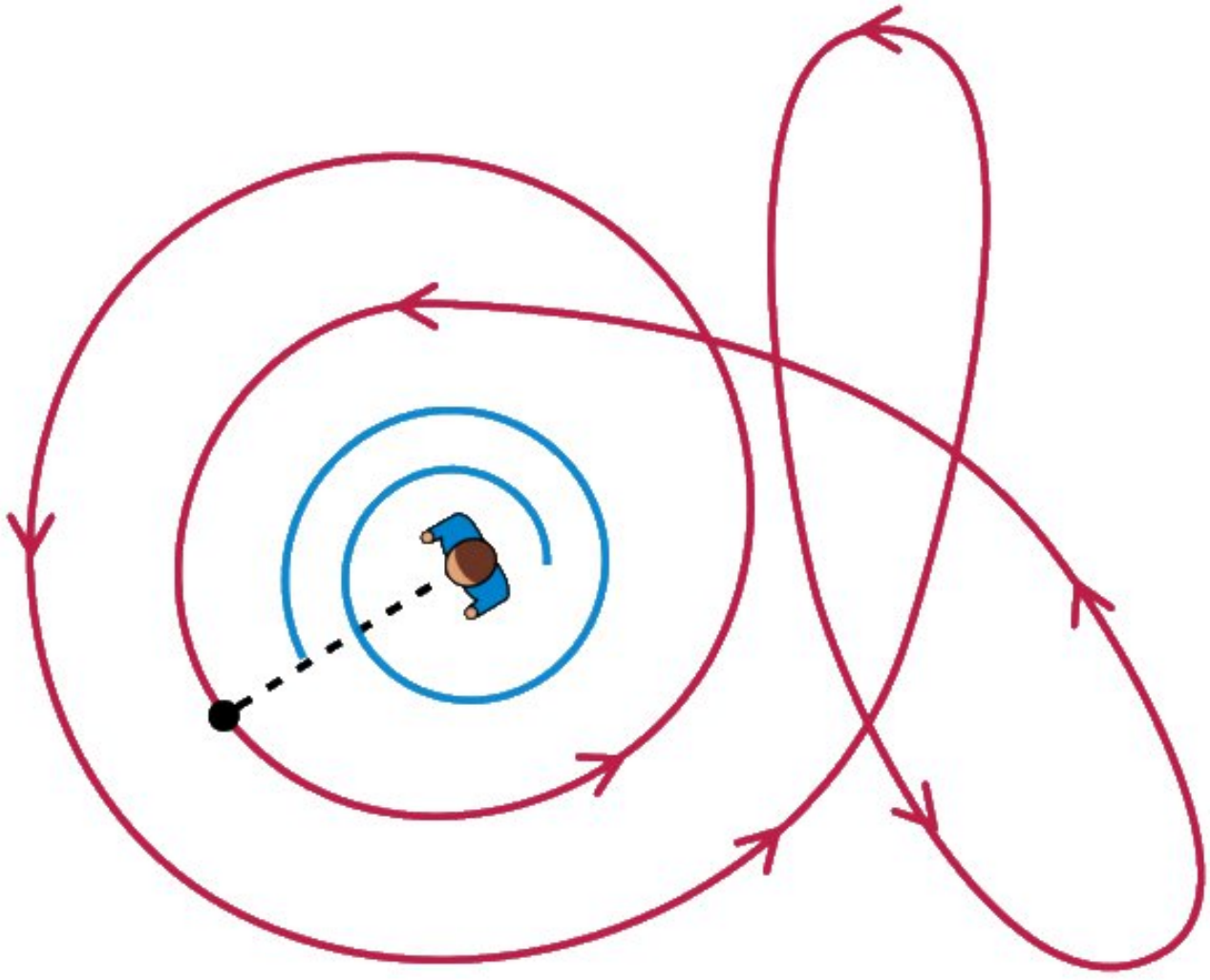

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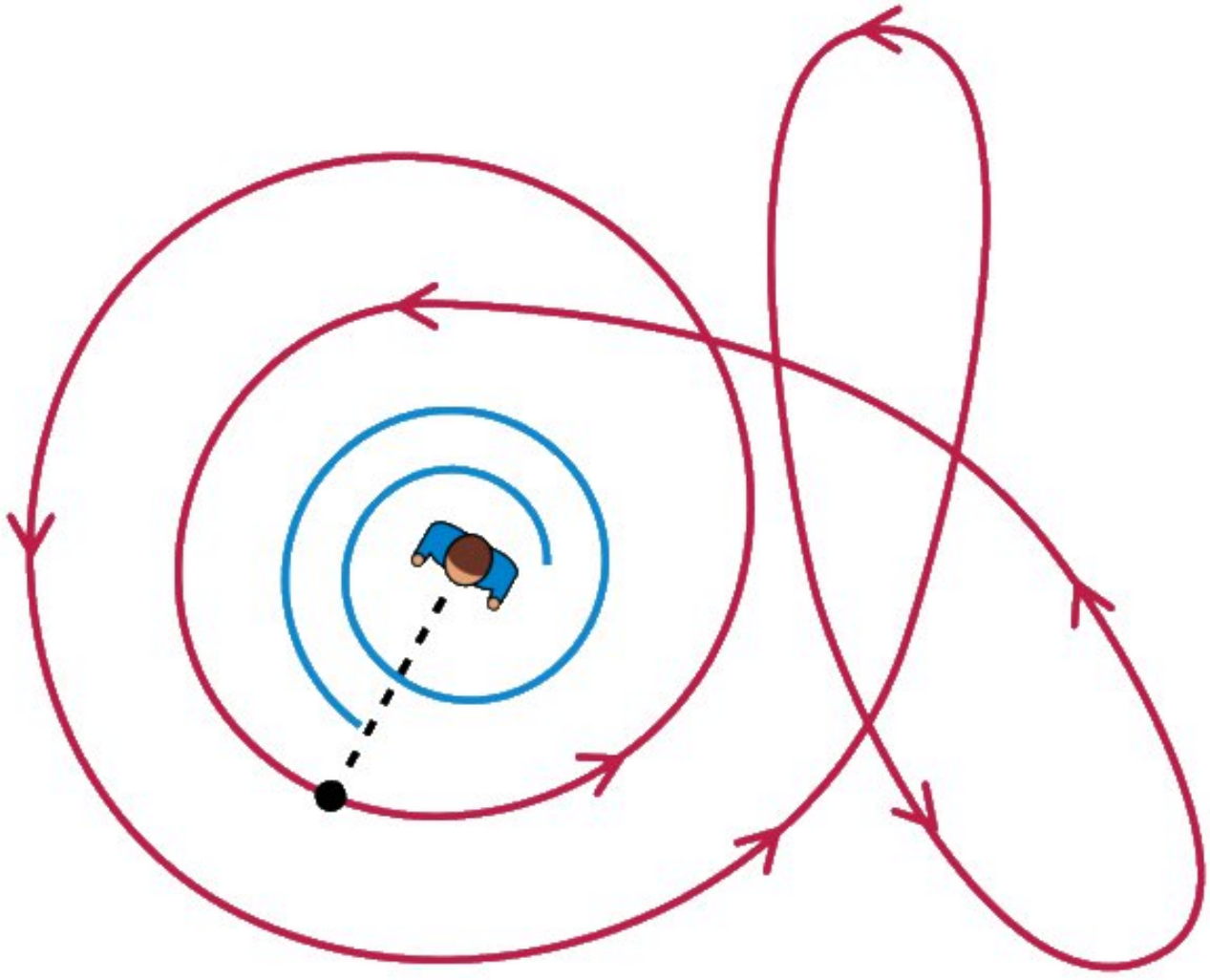

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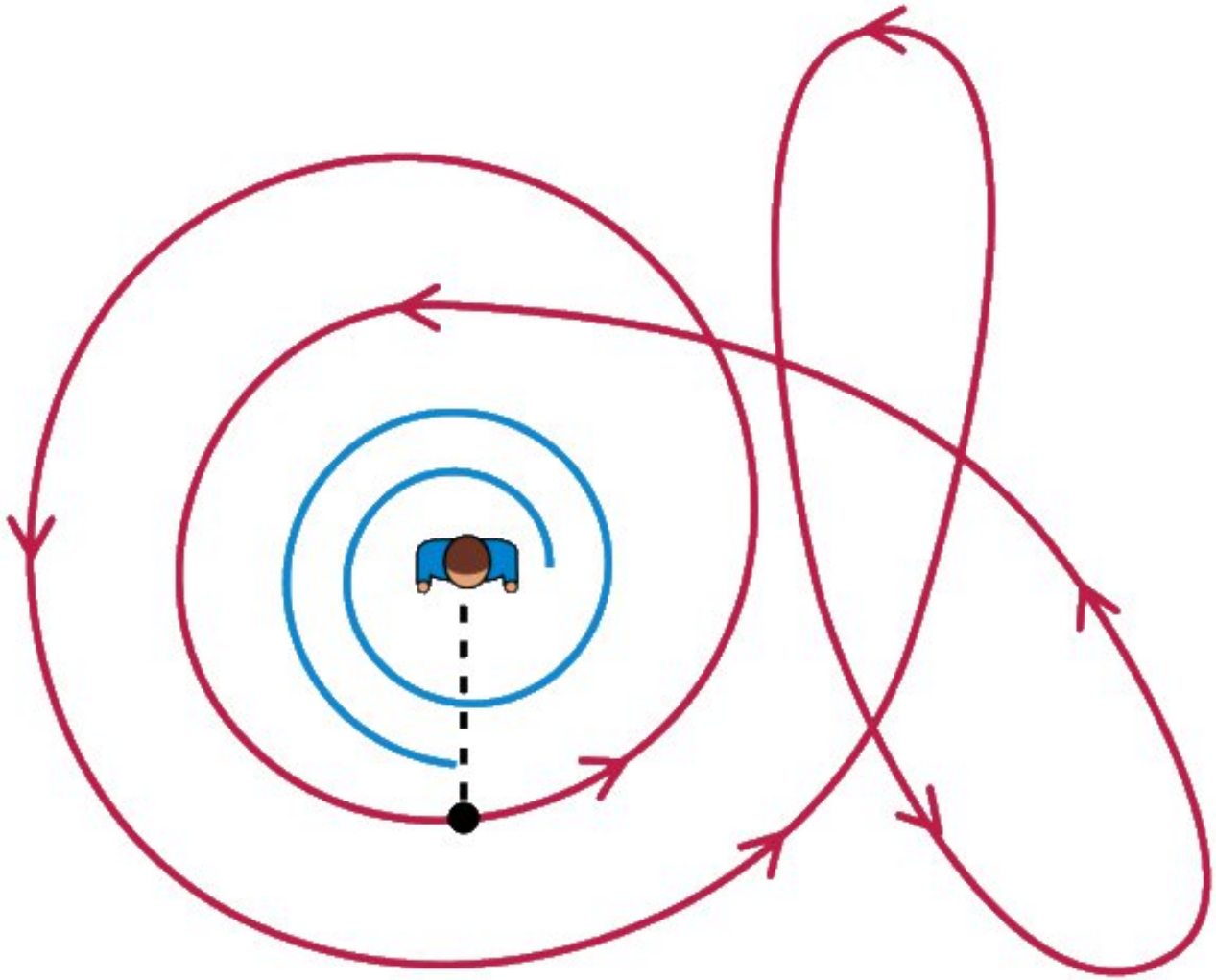
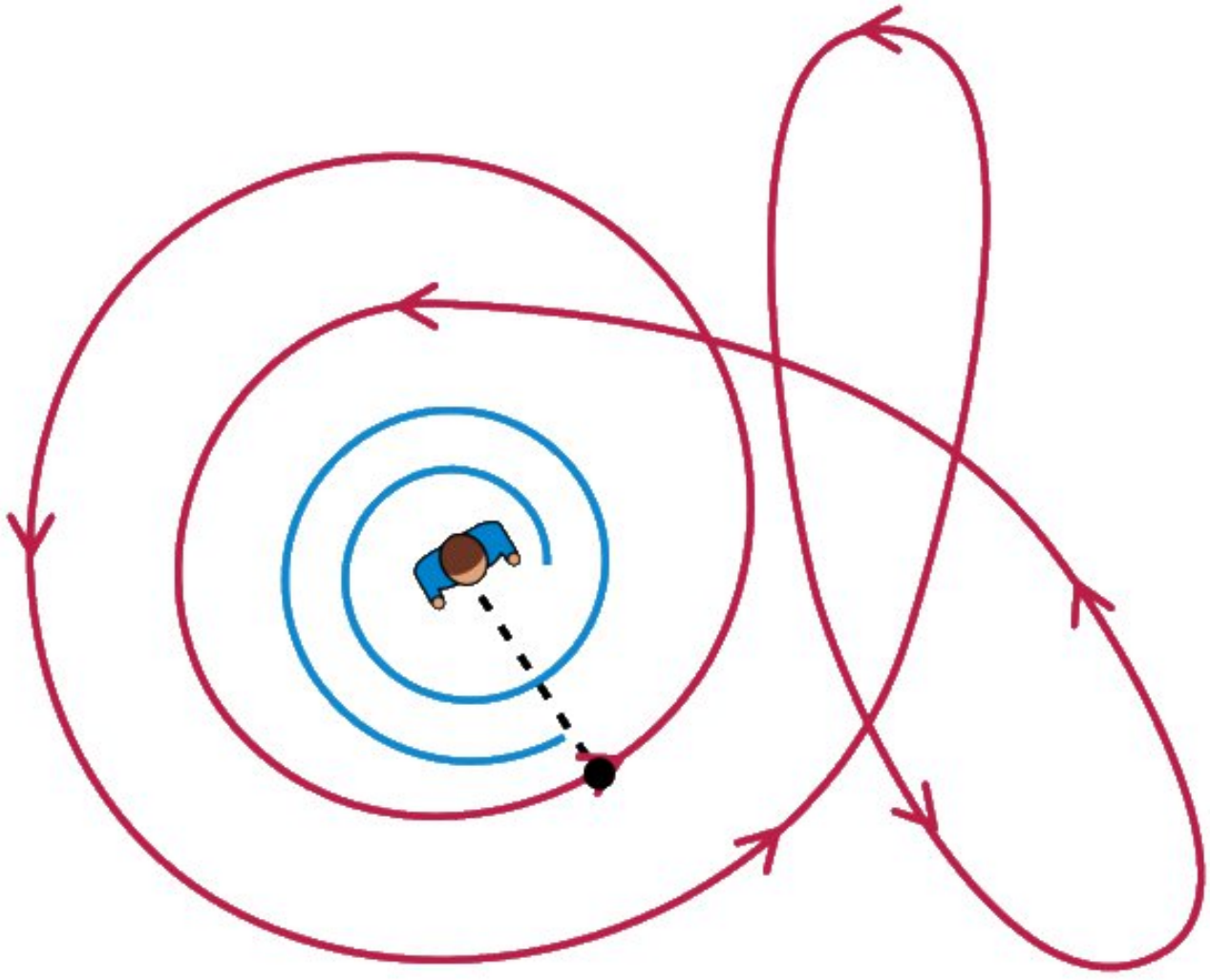
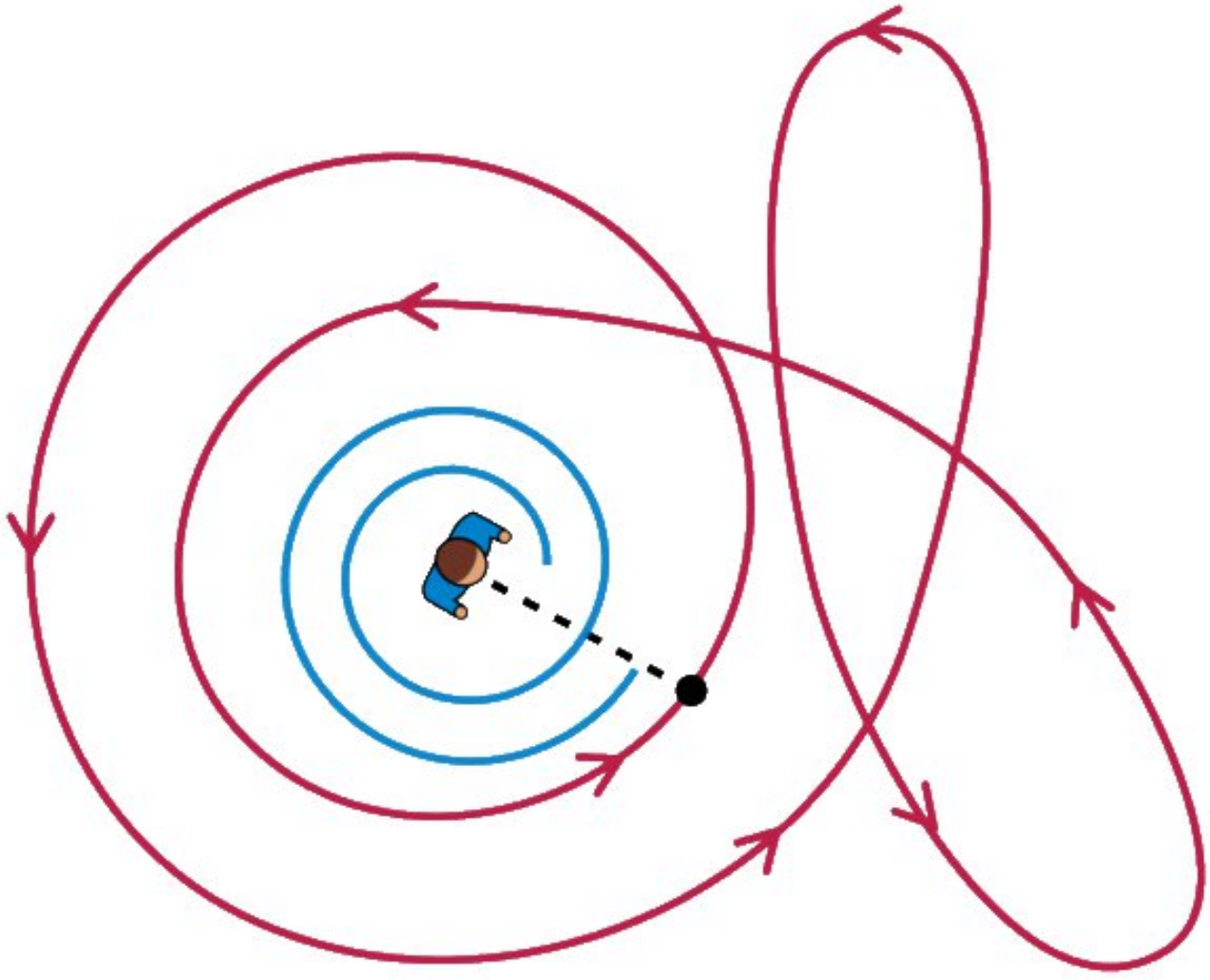


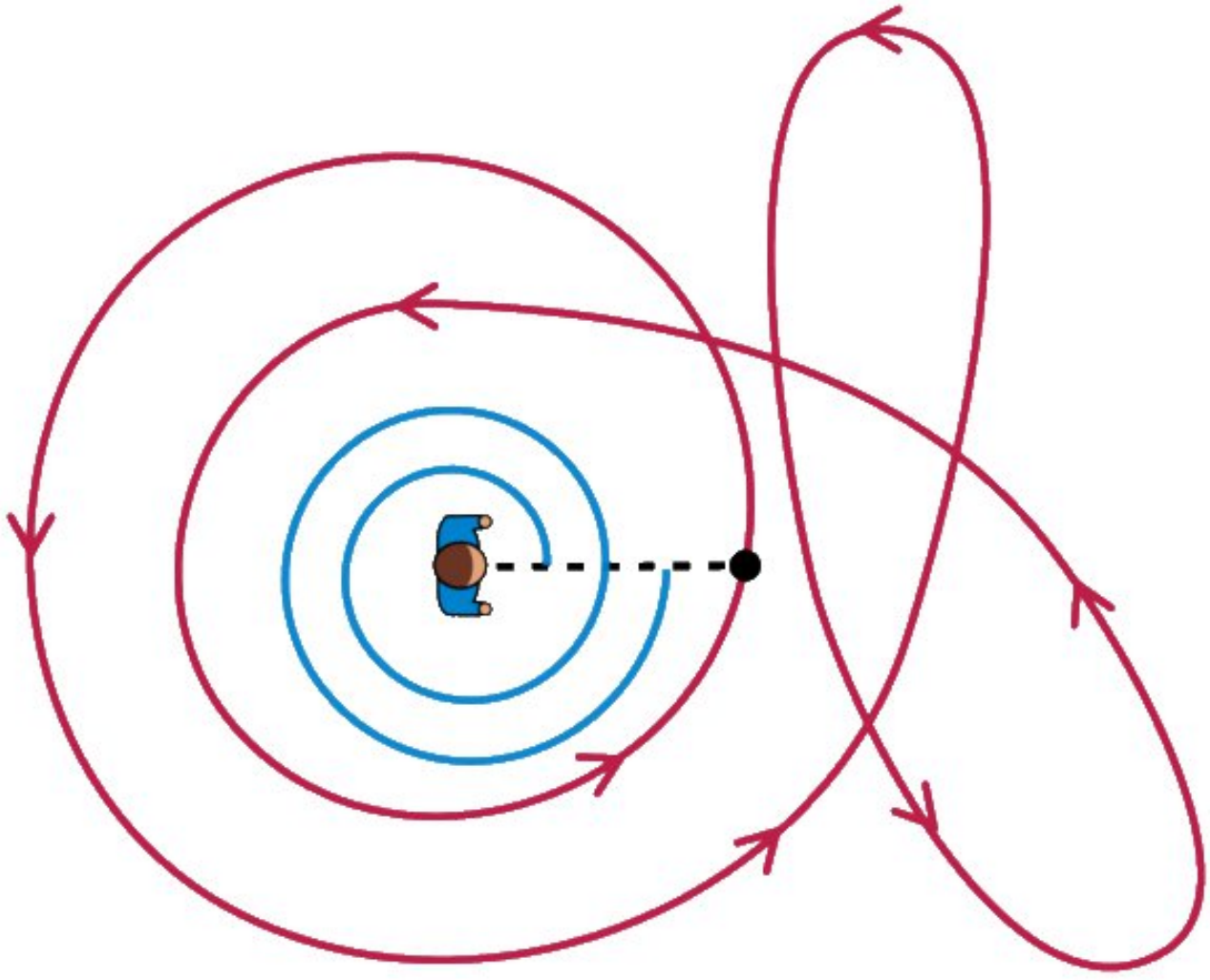
Winding numbers



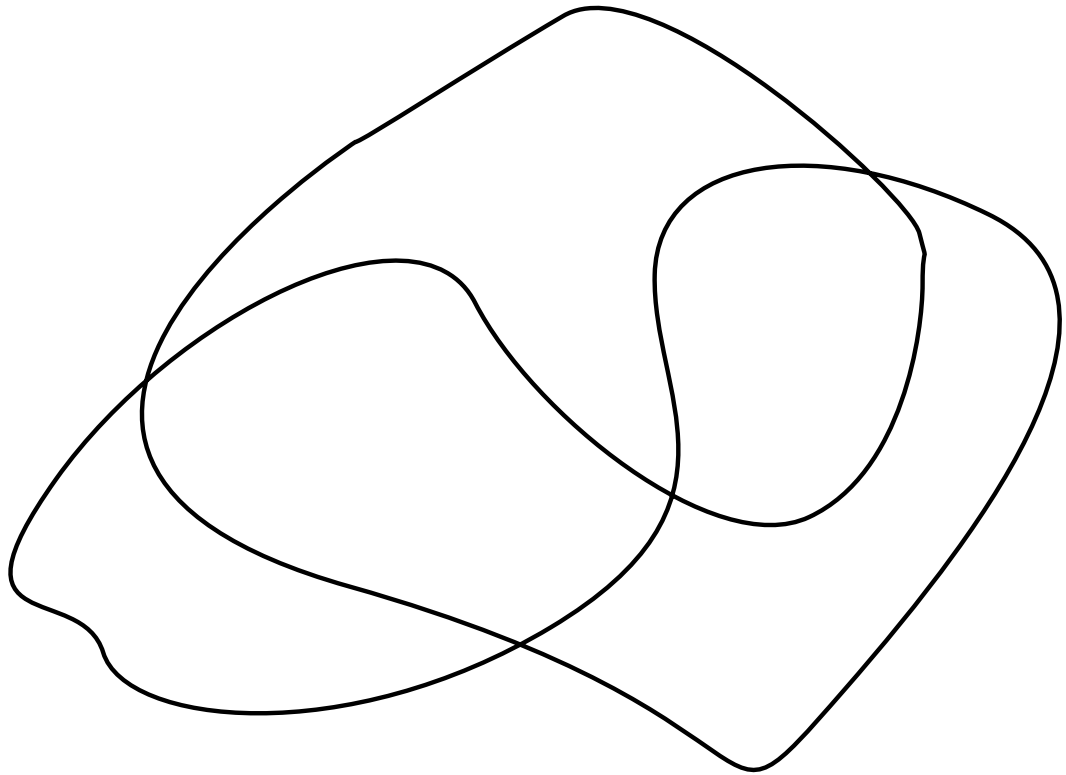
Winding numbers



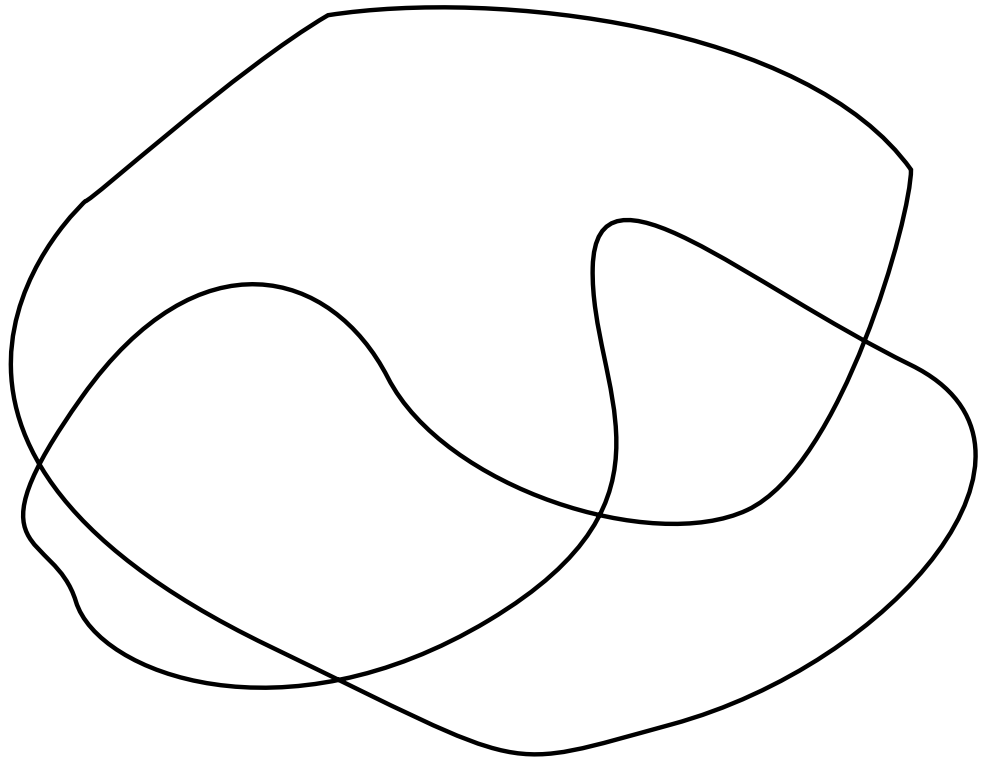
Winding numbers



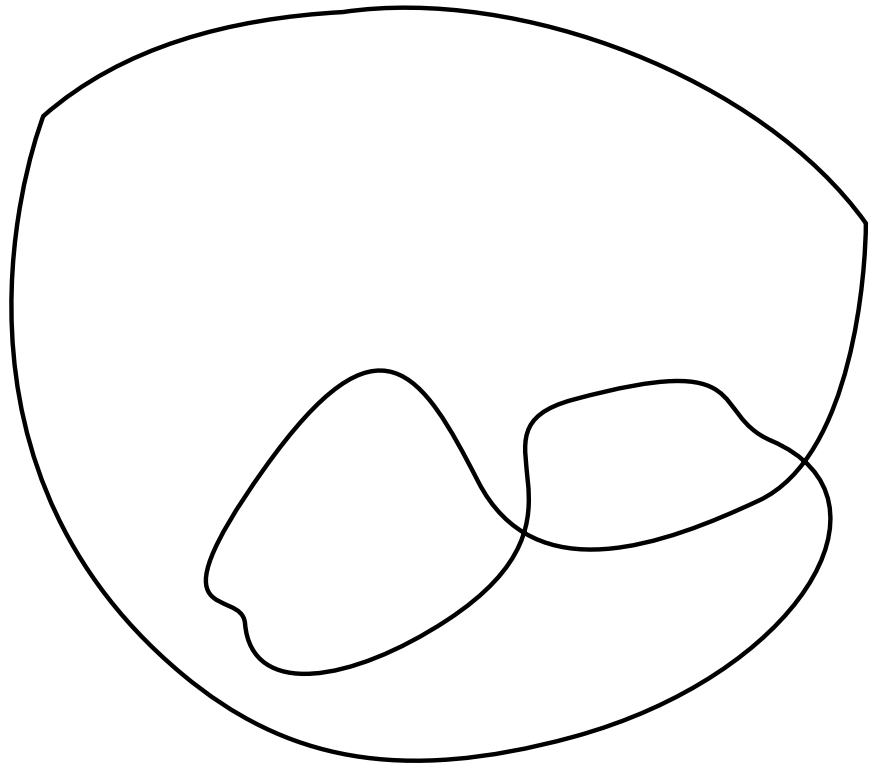
Deforming loops



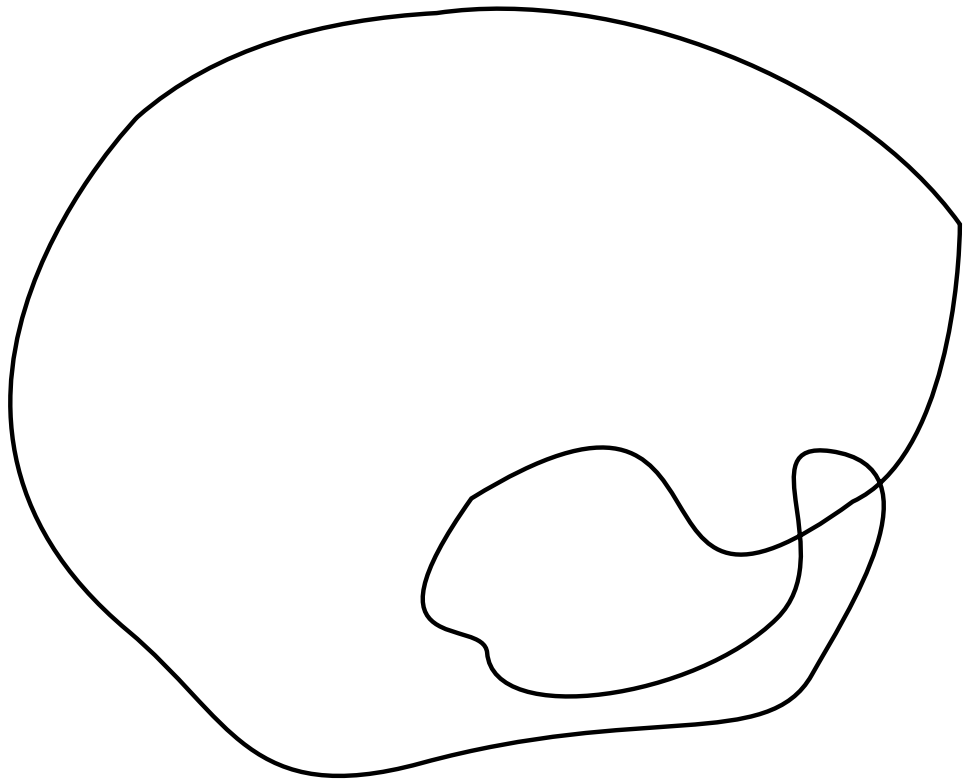
Deforming loops



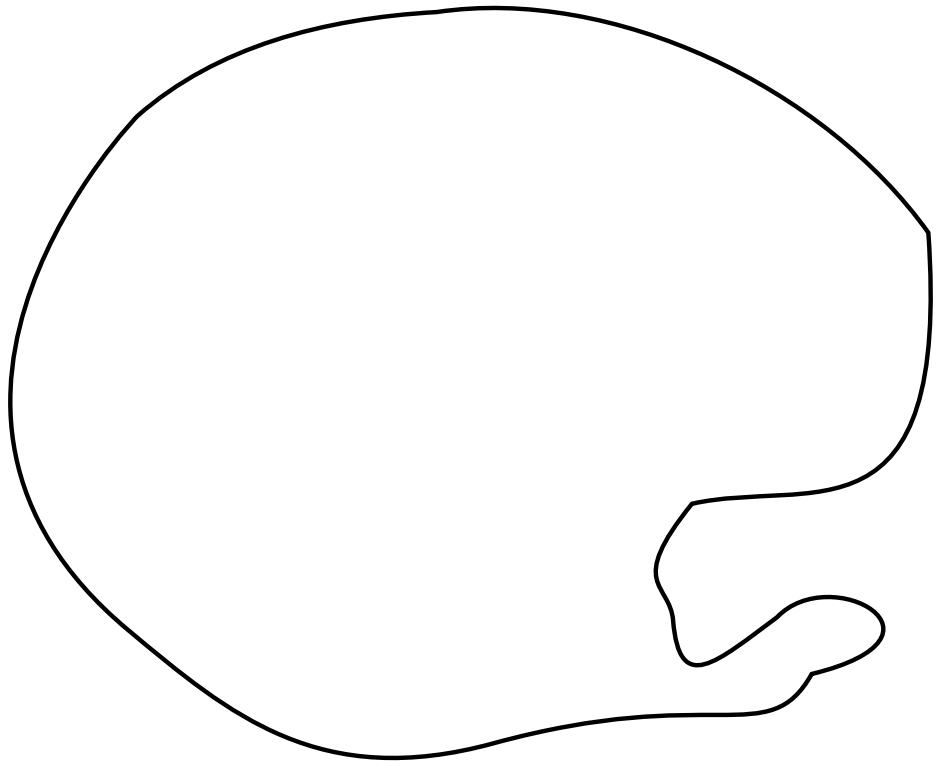
Deforming loops



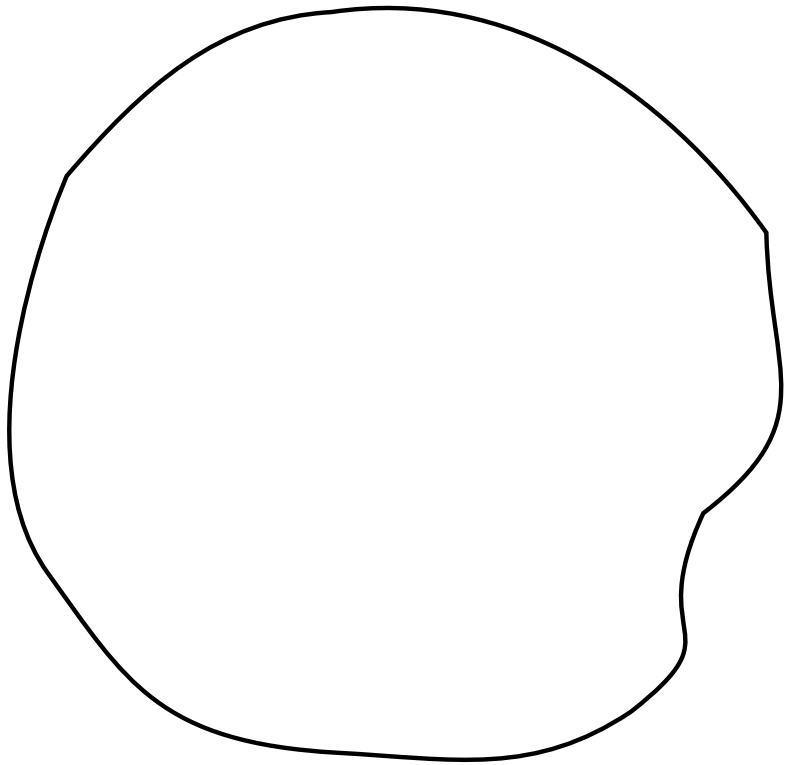
Deforming loops



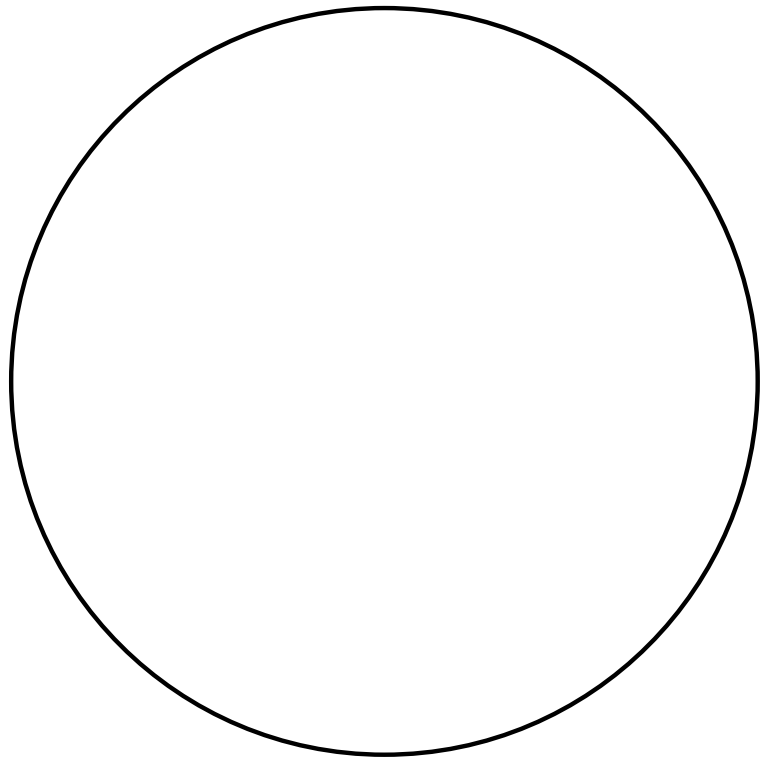
Deforming loops



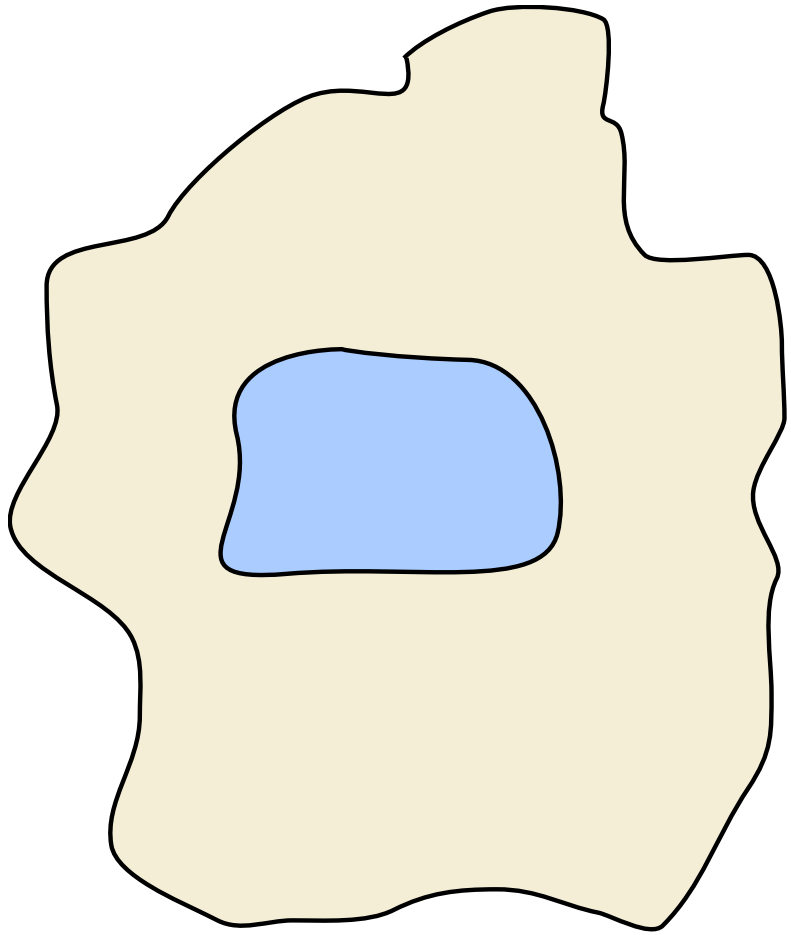
Deforming loops



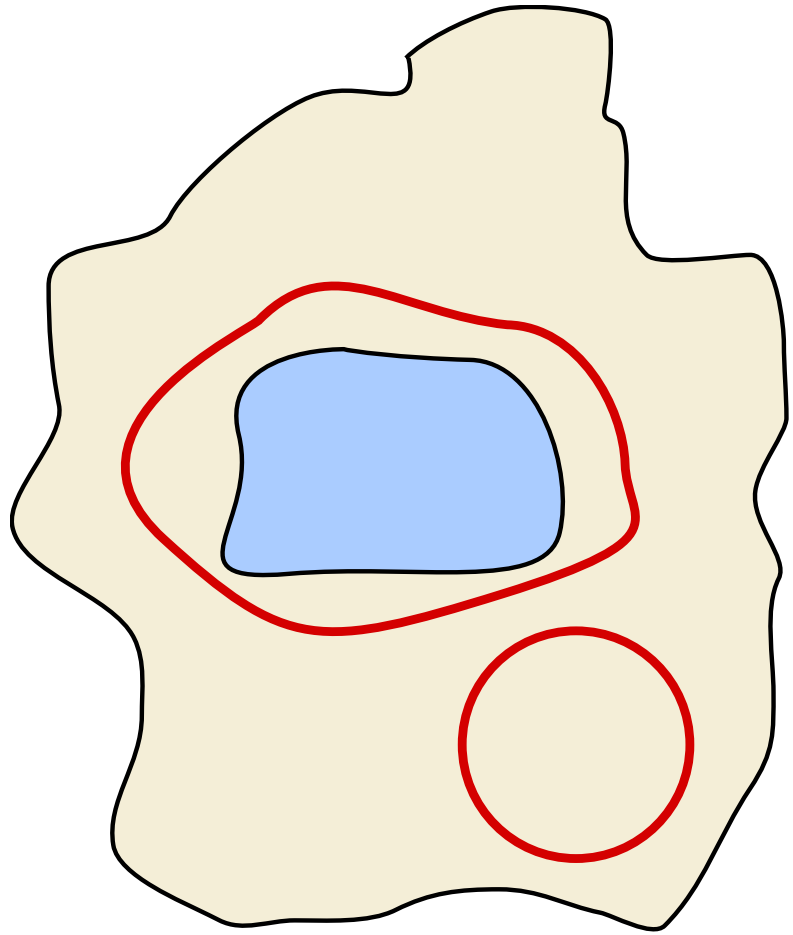
Deforming loops



Deforming loop with obstacles



Deforming loop with obstacles



Hopf's degree theorem

Hopf's degree theorem

Theorem.

Hopf's degree theorem

Theorem. A loop γ_1

Hopf's degree theorem

Theorem. A loop γ_1 can be deformed on to another loop γ_2

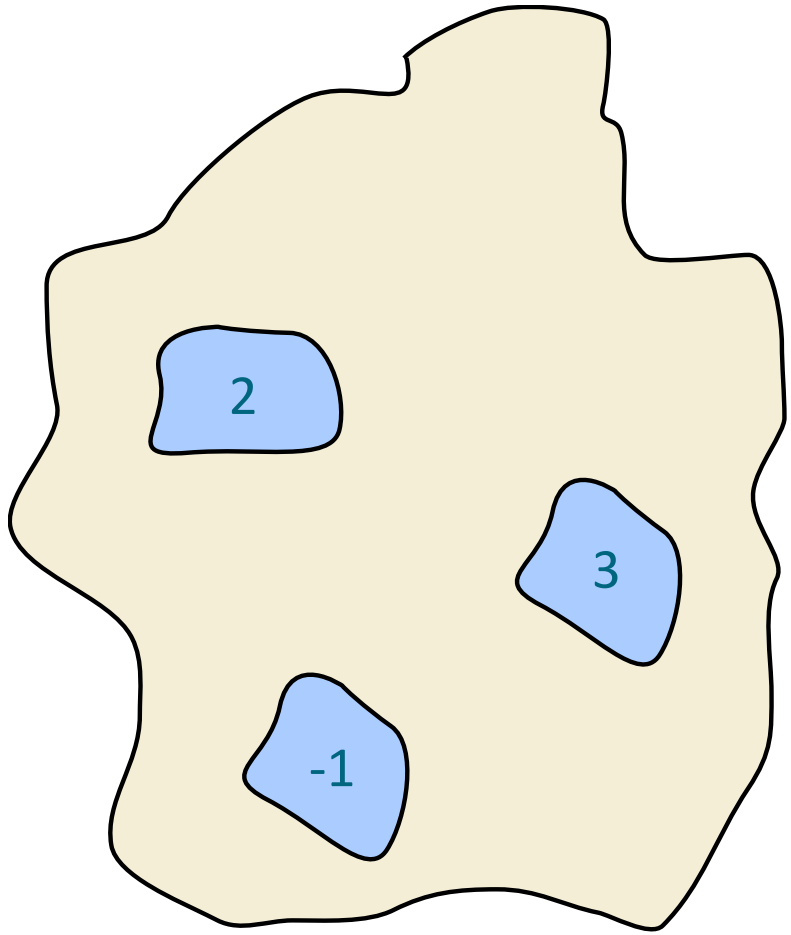
Hopf's degree theorem

Theorem. A loop γ_1 can be deformed on to another loop γ_2 whilst avoiding the obstacle p

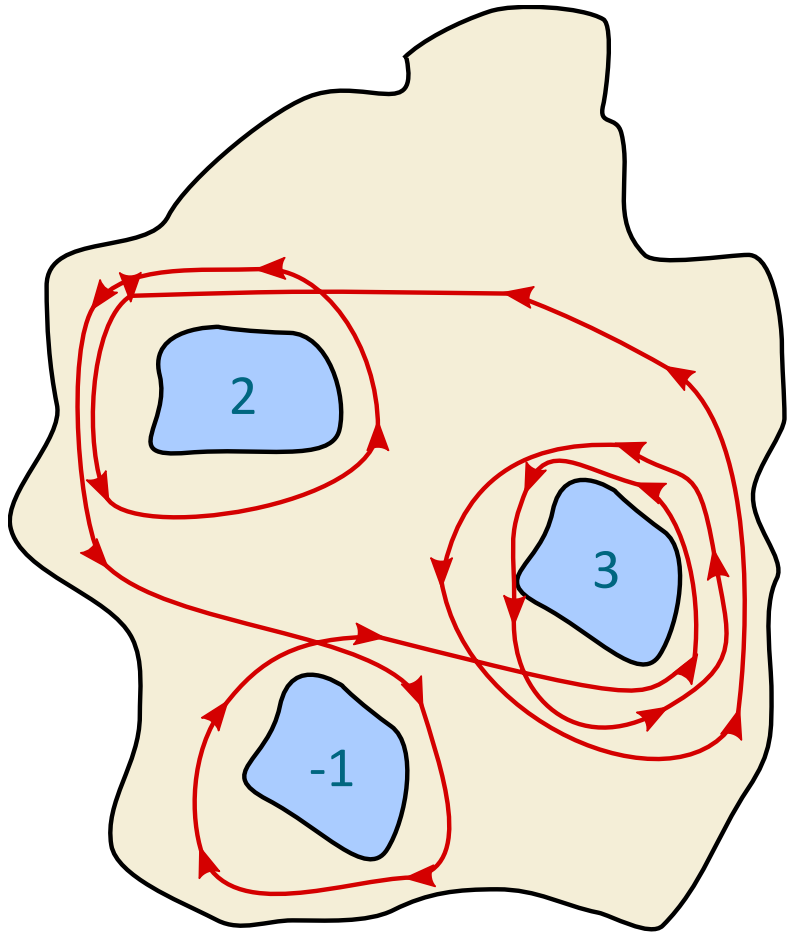
Hopf's degree theorem

Theorem. A loop γ_1 can be deformed on to another loop γ_2 whilst avoiding the obstacle p if and only if γ_1 and γ_2 have the same winding number about p.

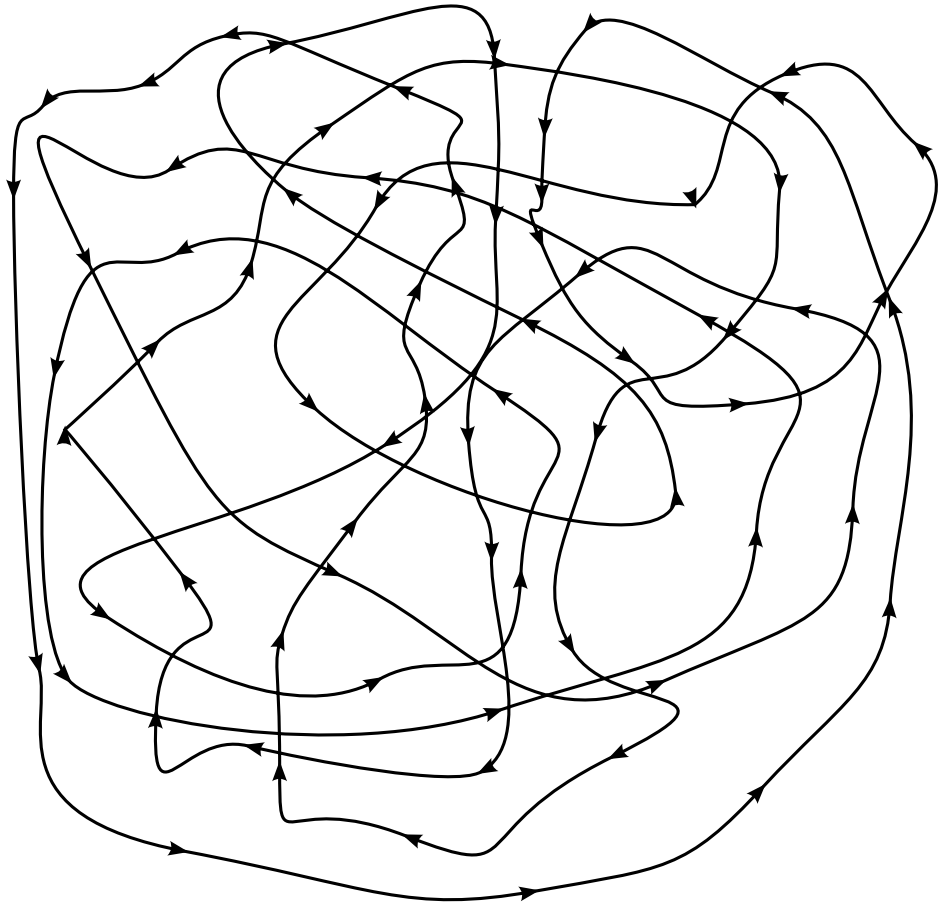
Connectivity using winding numbers



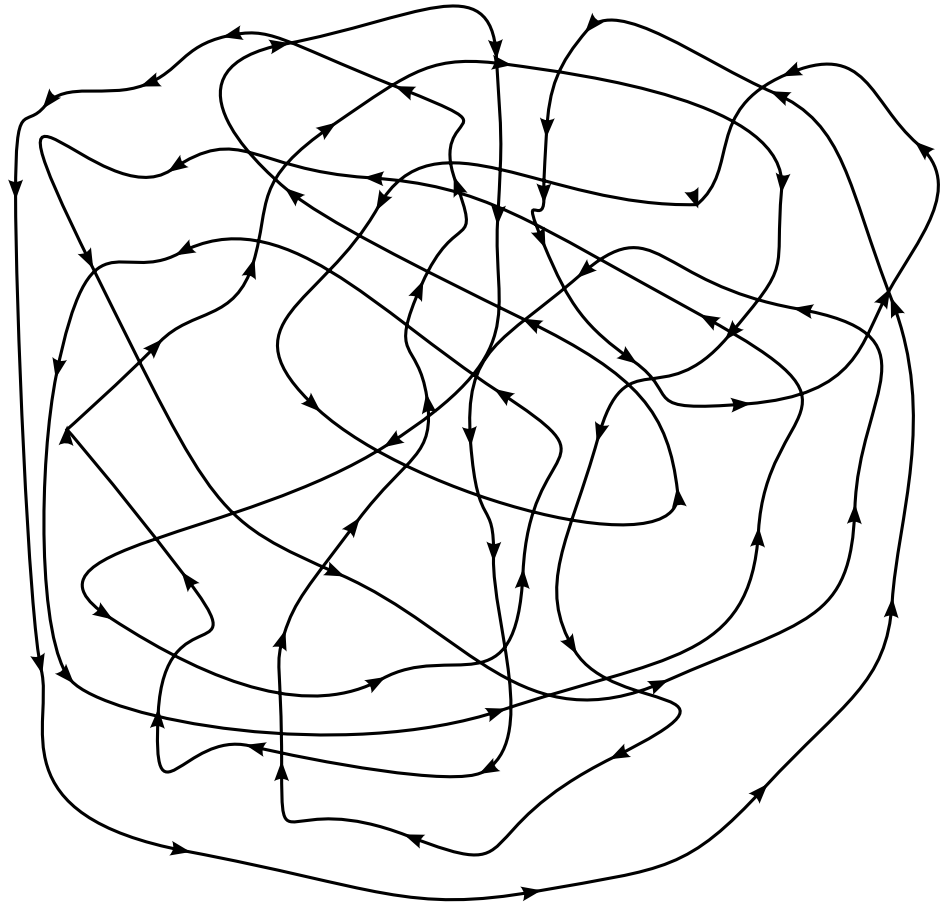
Connectivity using winding numbers



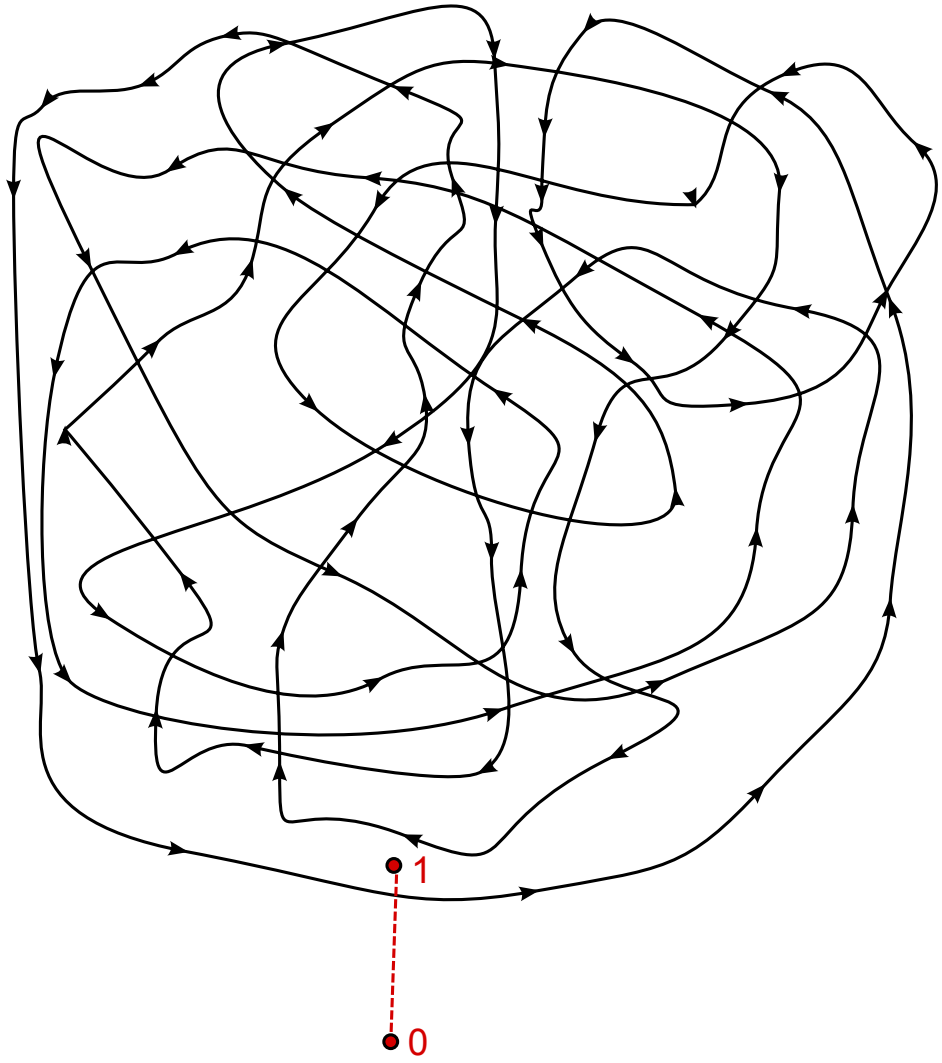
How to calculate winding numbers



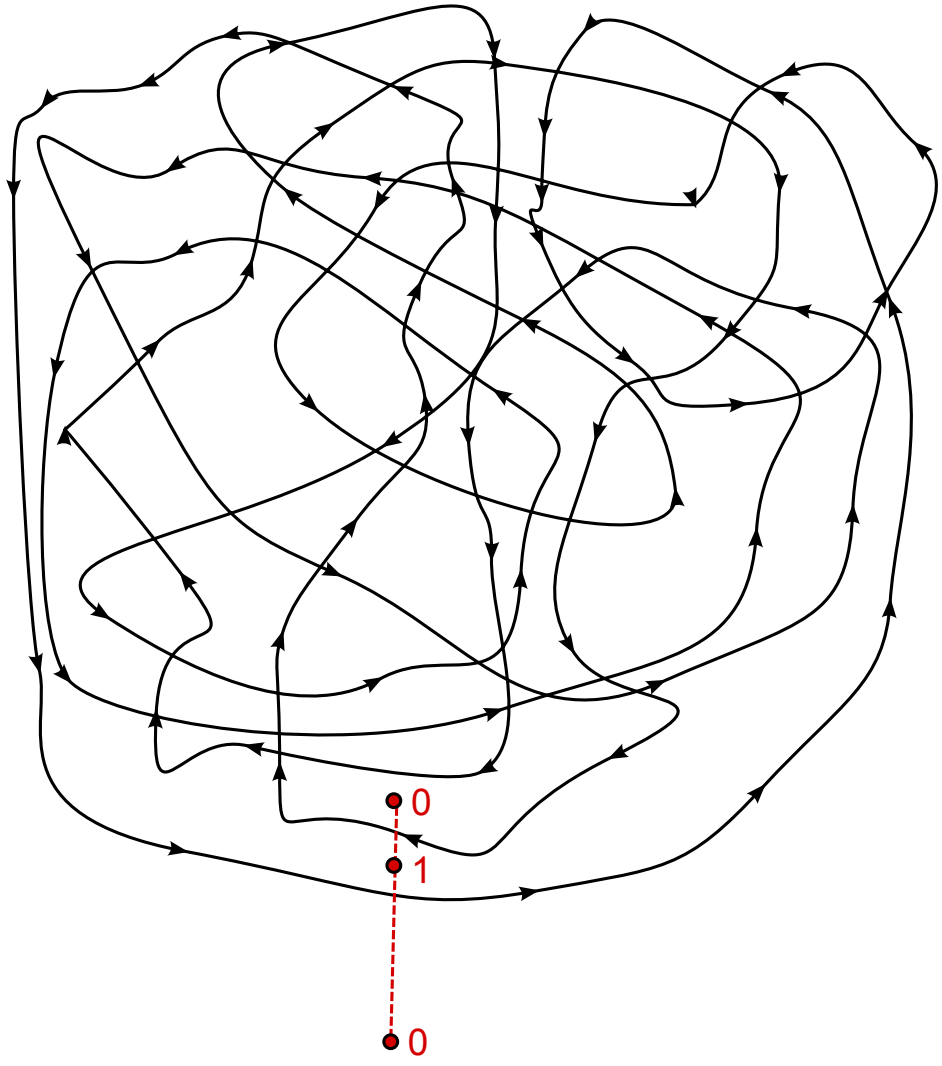
How to calculate winding numbers



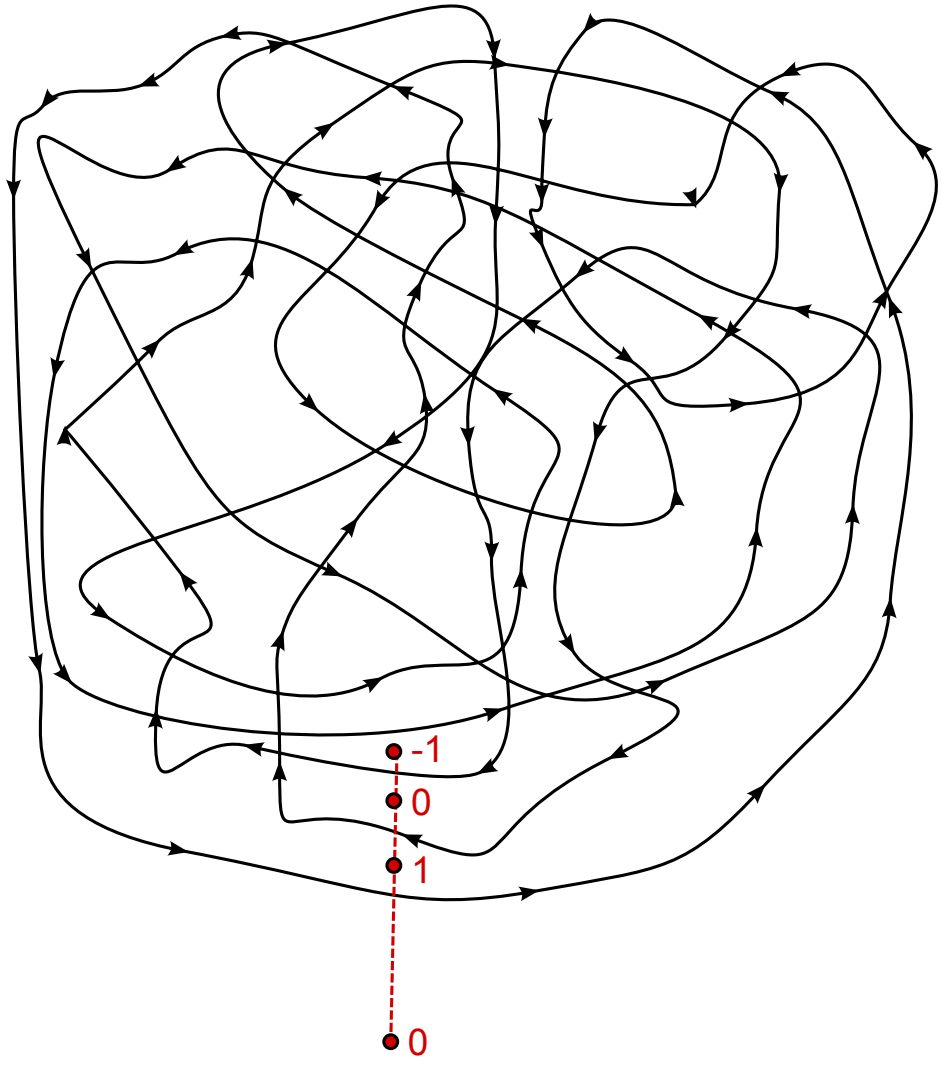
How to calculate winding numbers



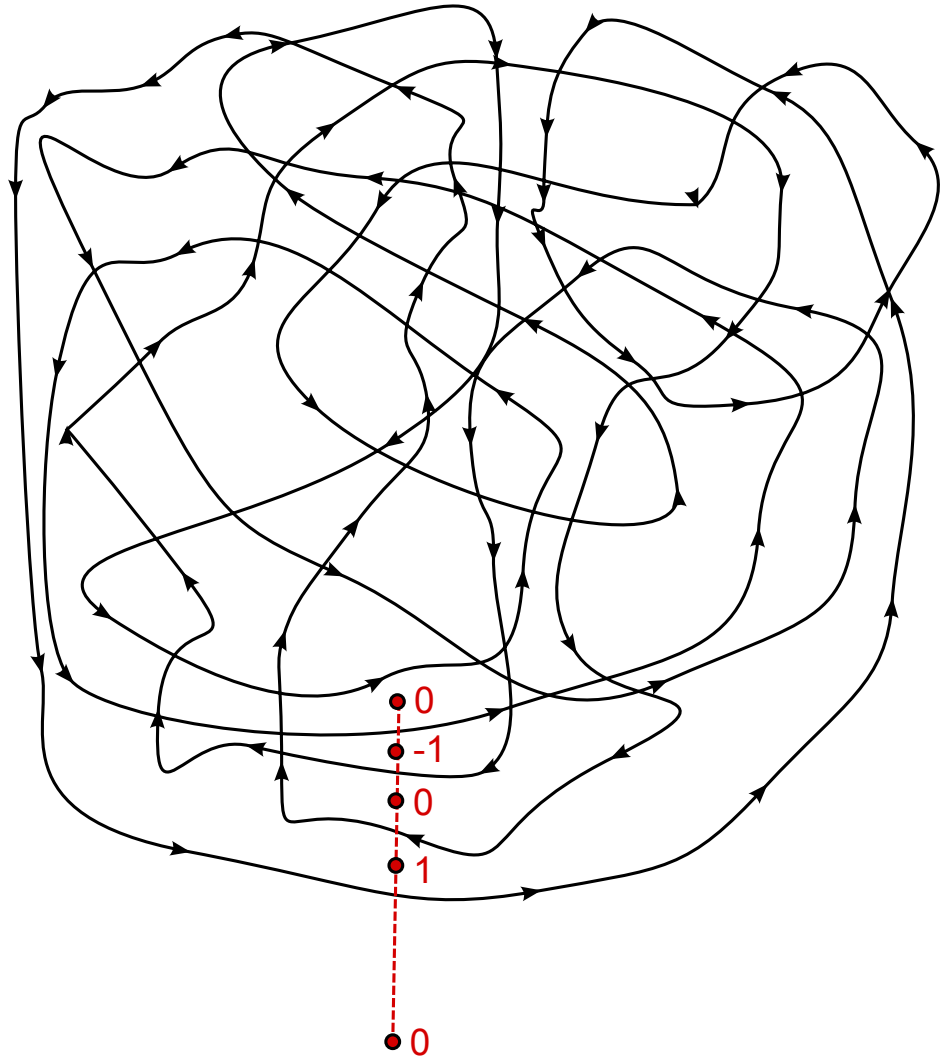
How to calculate winding numbers



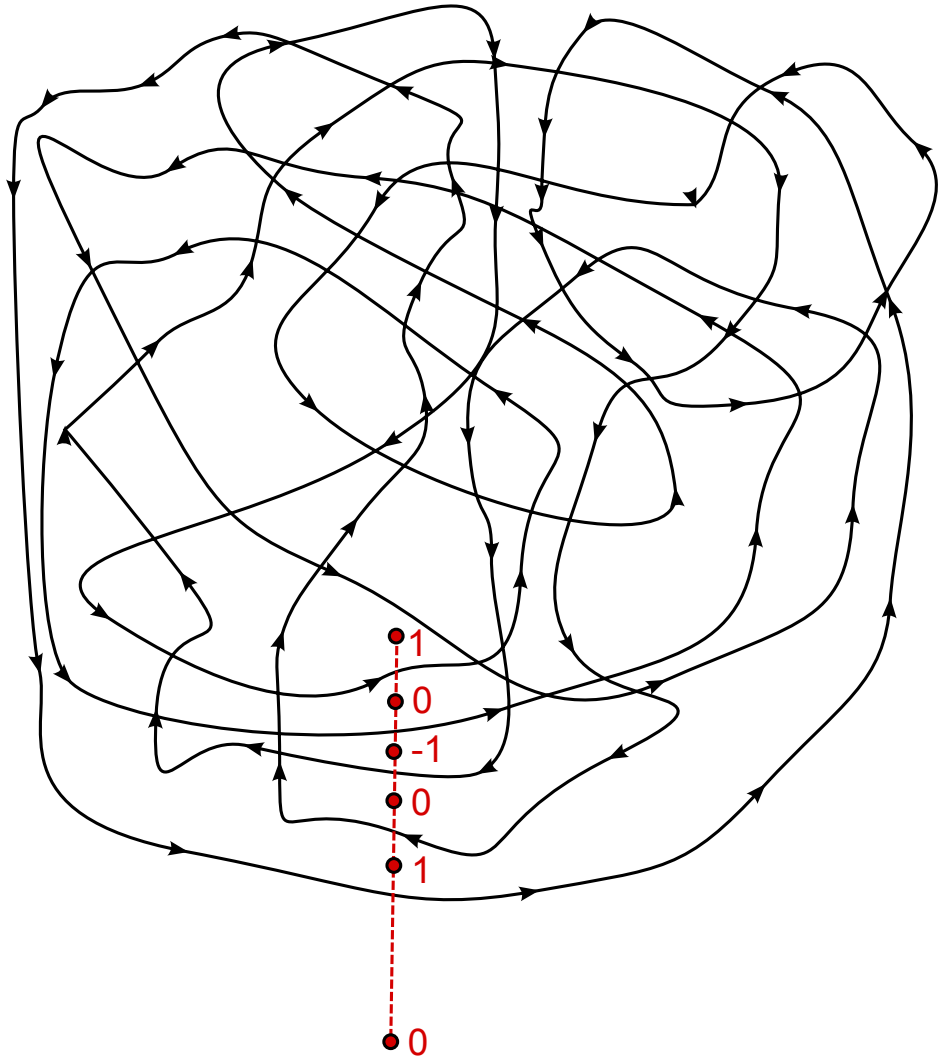
How to calculate winding numbers



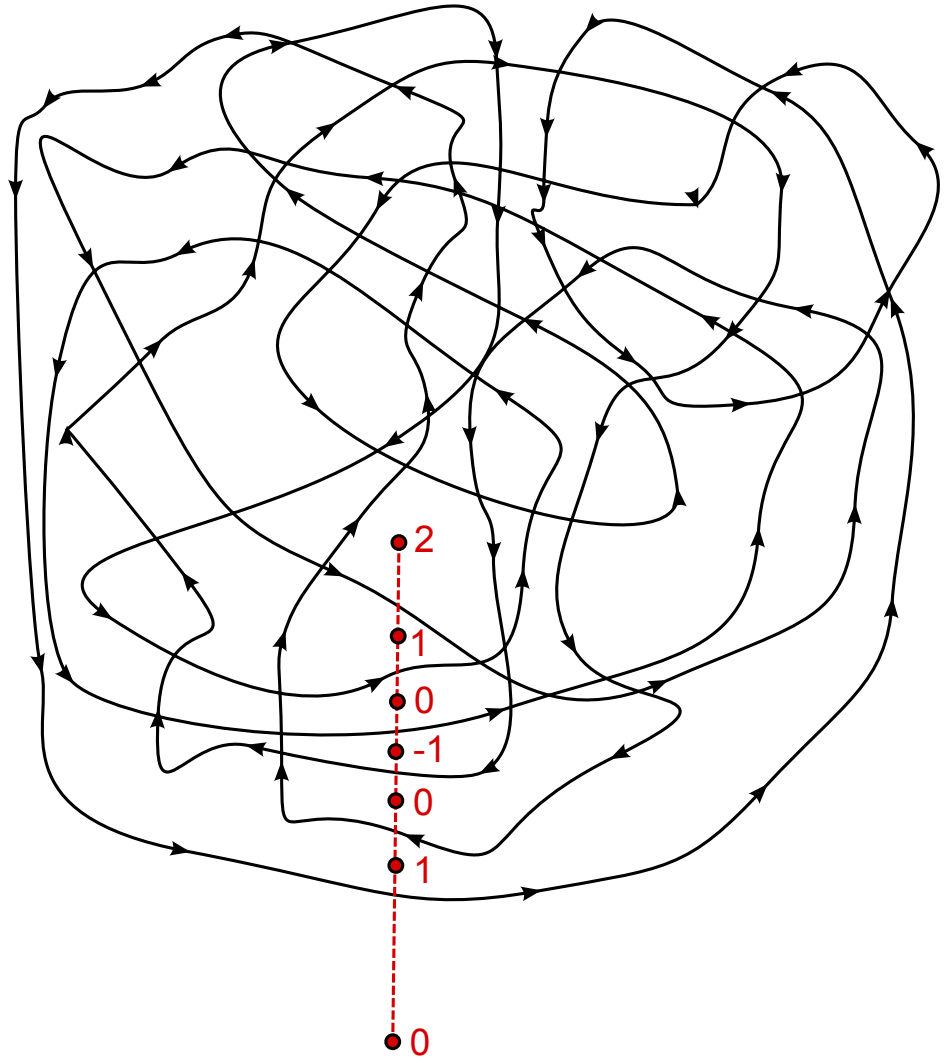
How to calculate winding numbers



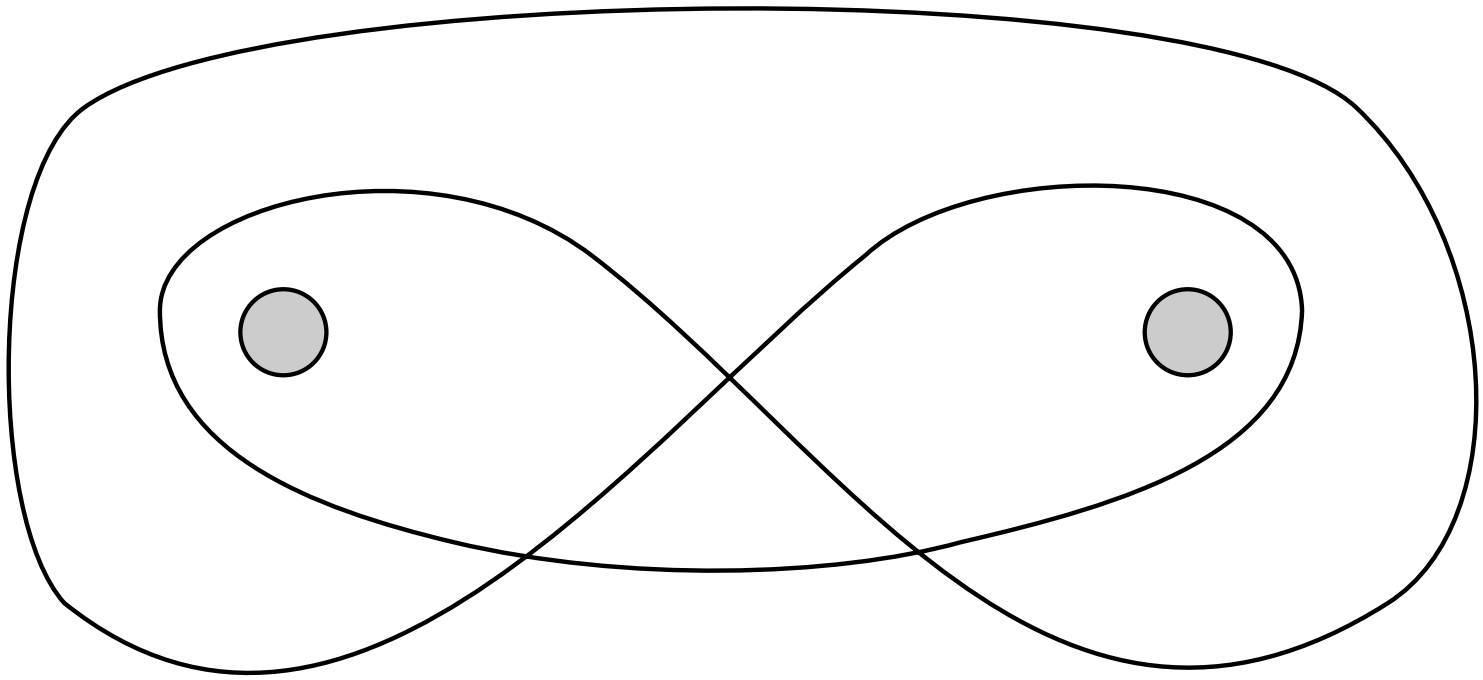
How to calculate winding numbers



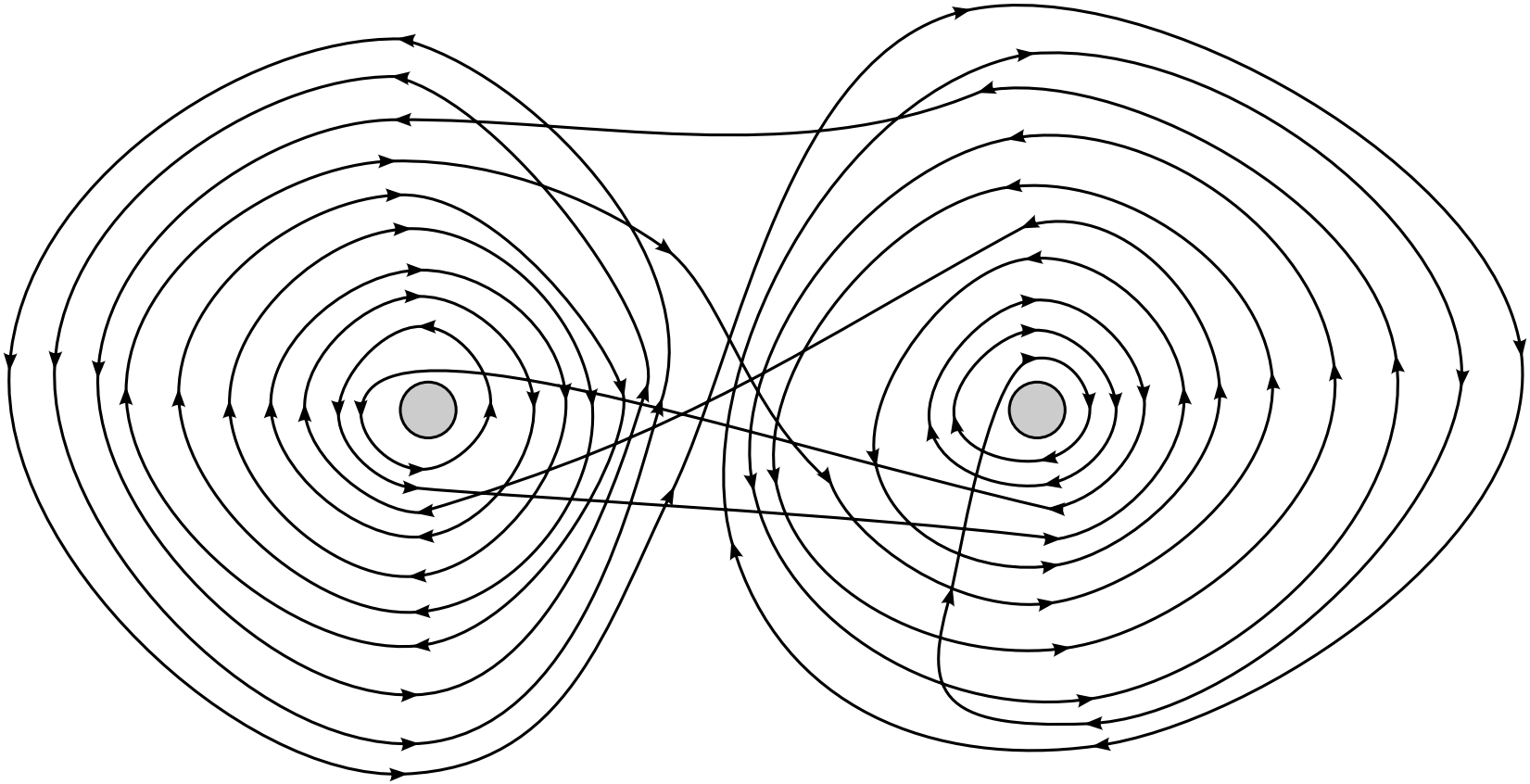
How to calculate winding numbers



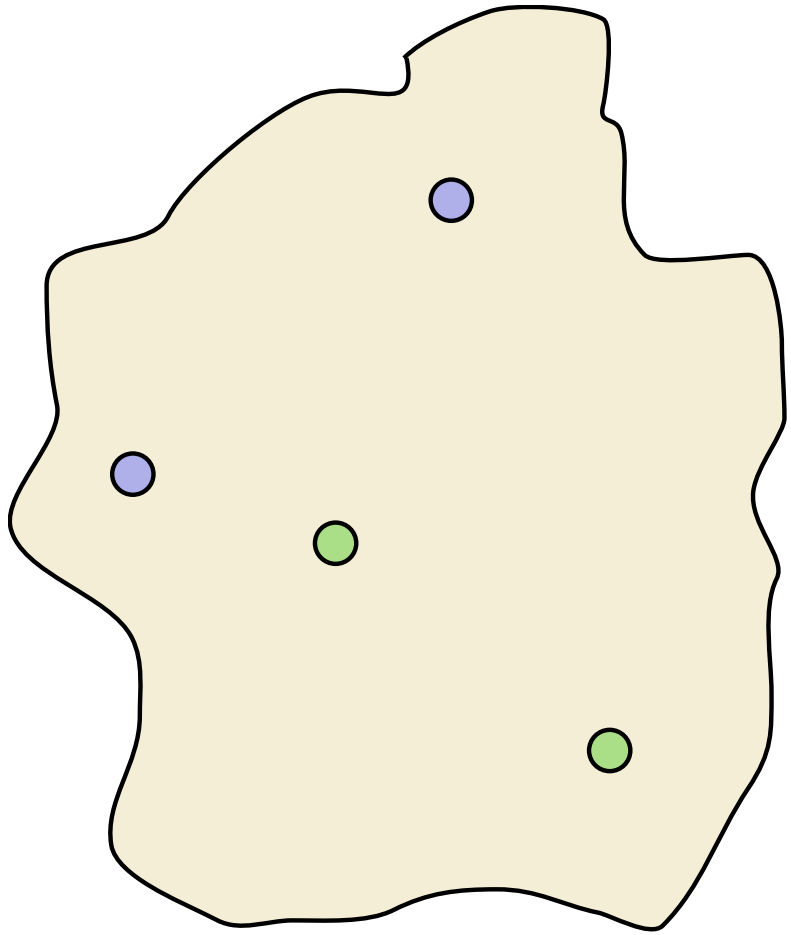
Hanging problem



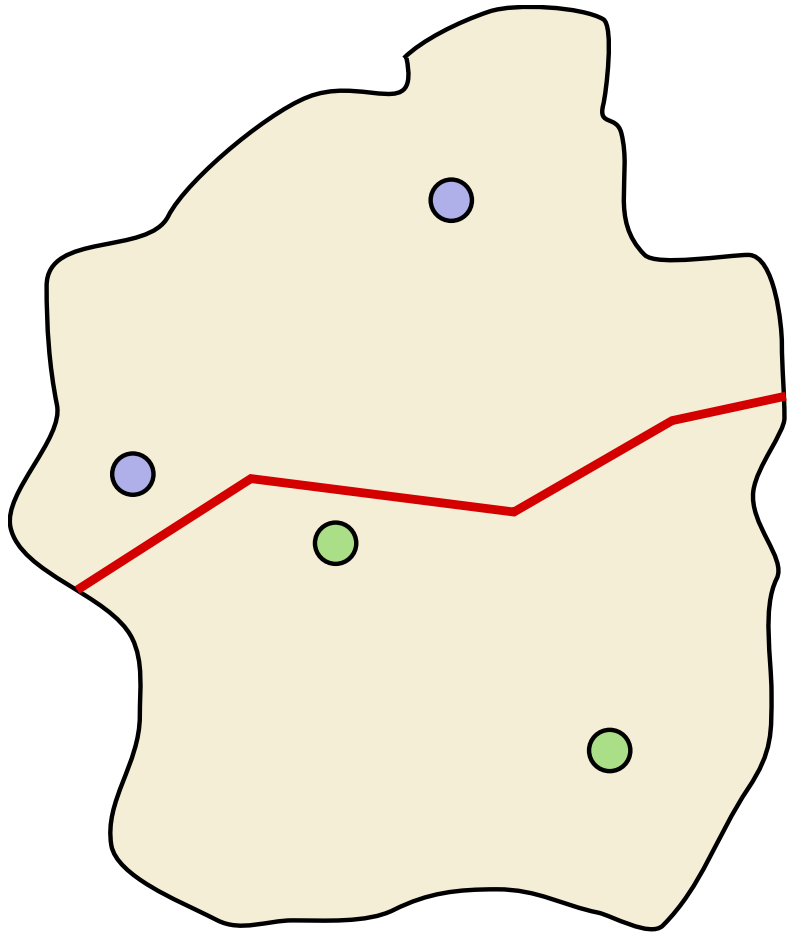
Hanging problem



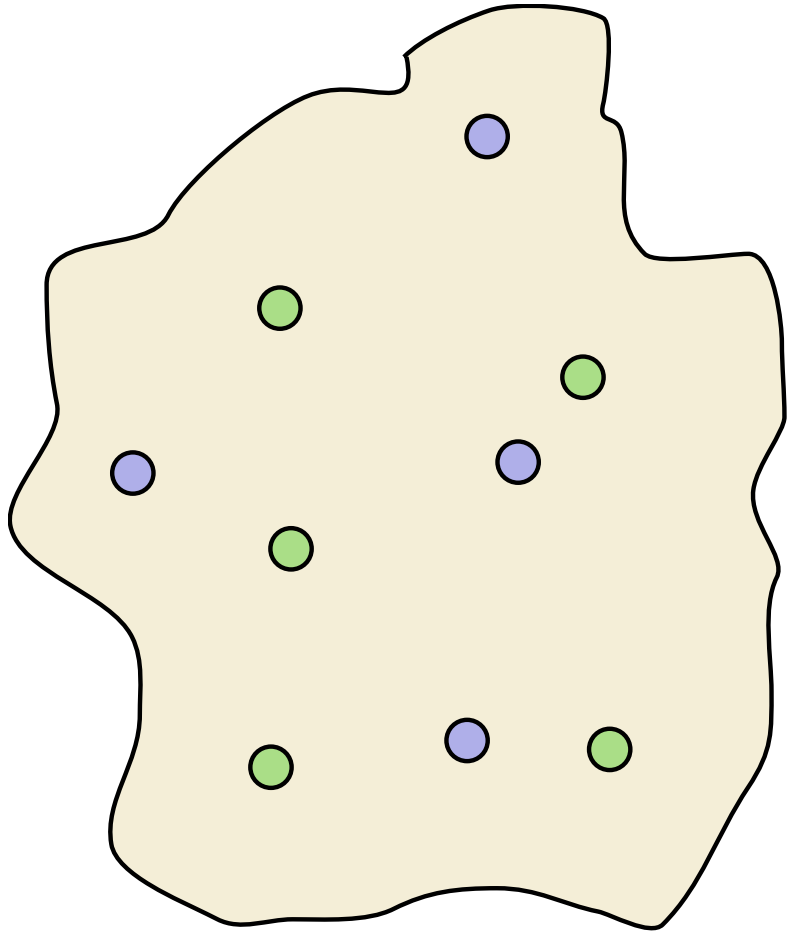
Dividing walls



Dividing walls



Problem

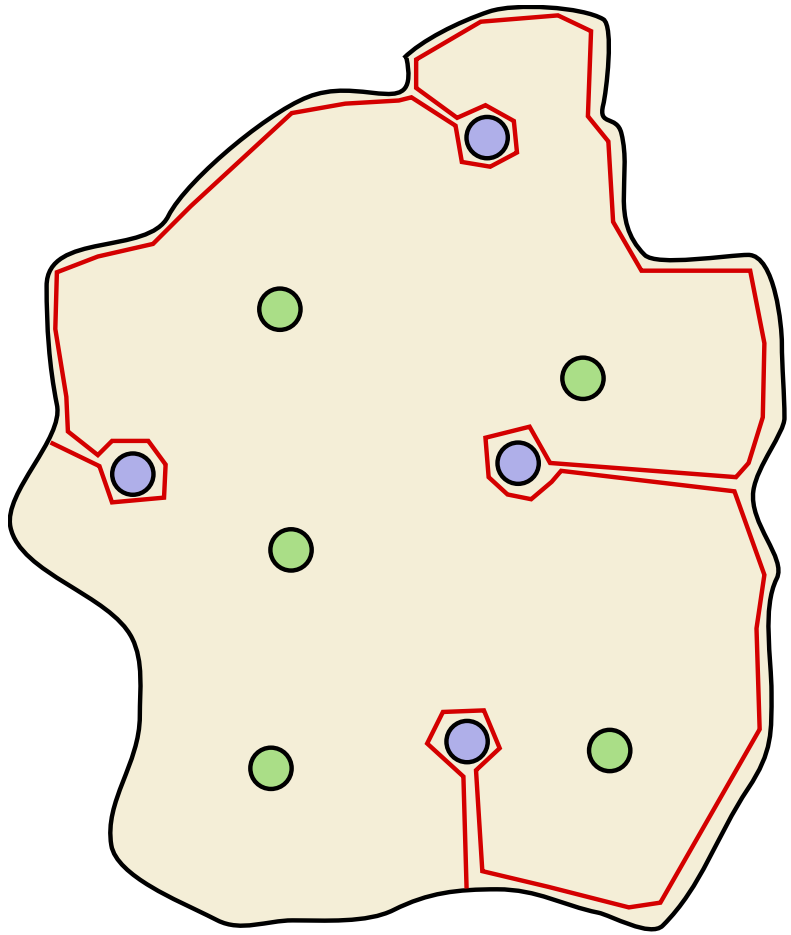


Is there a dividing wall for this set of towns?

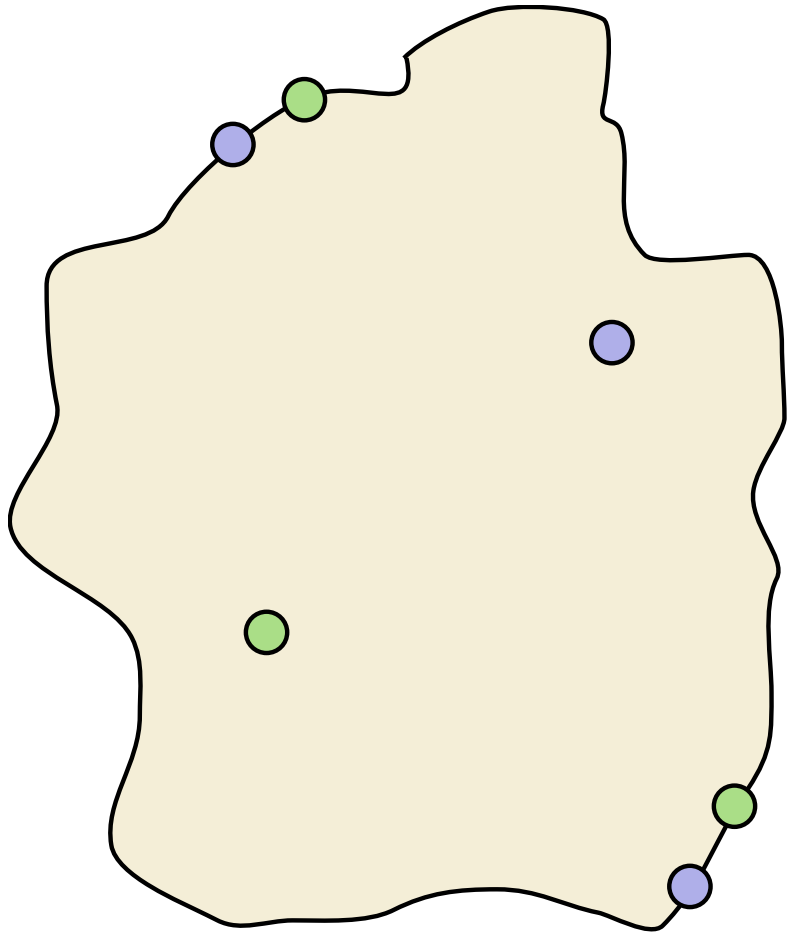
Experiment with other configurations of towns.

A general strategy

A general strategy

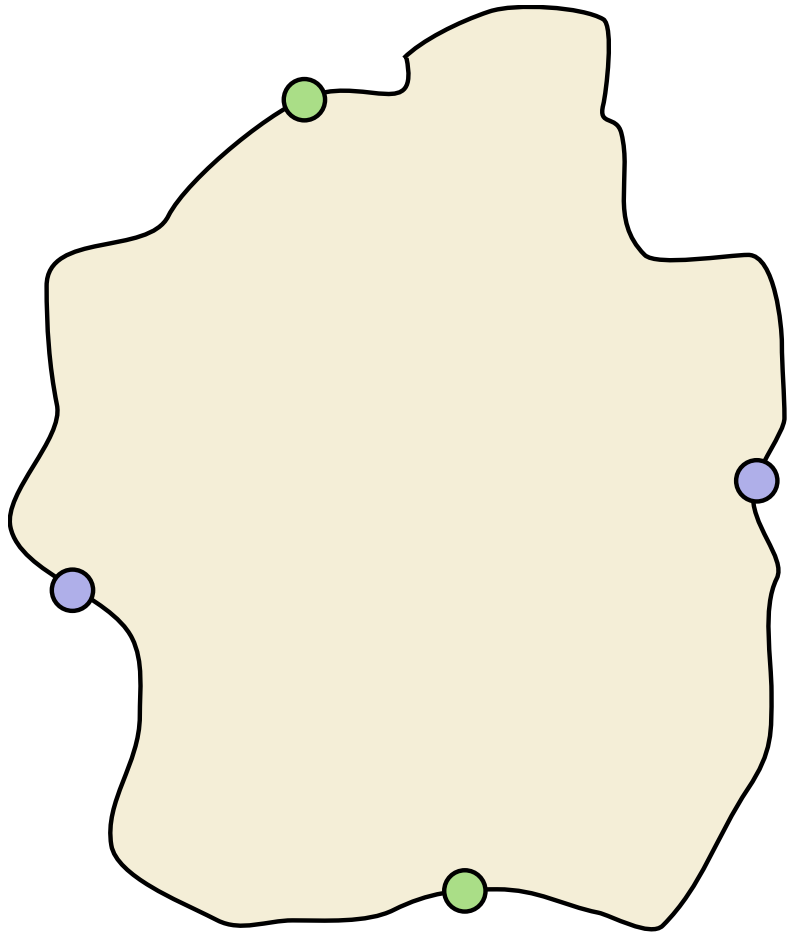


Coastal towns

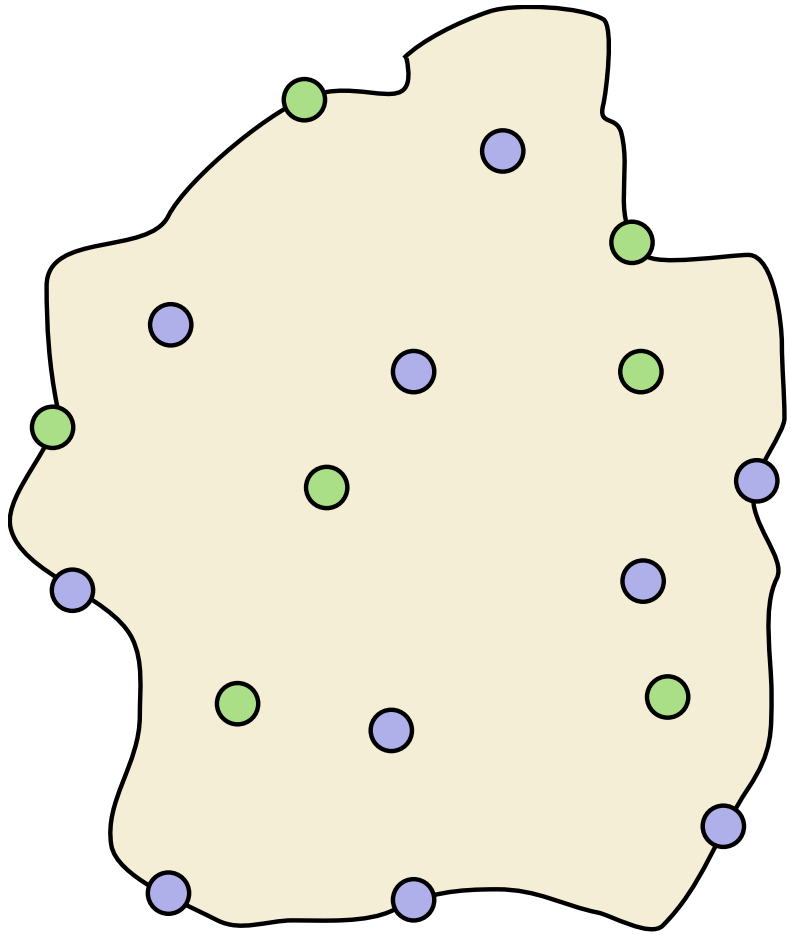


Experiment with other configurations of coastal towns and inland towns.

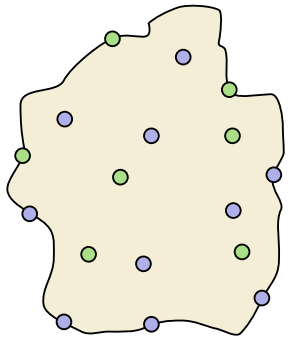
Simplist problematic configuration



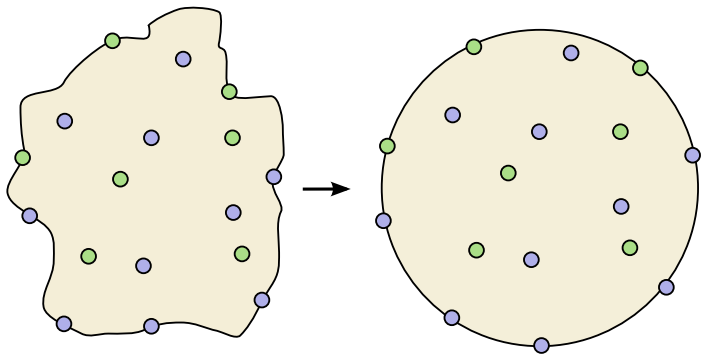
Discard the problem case



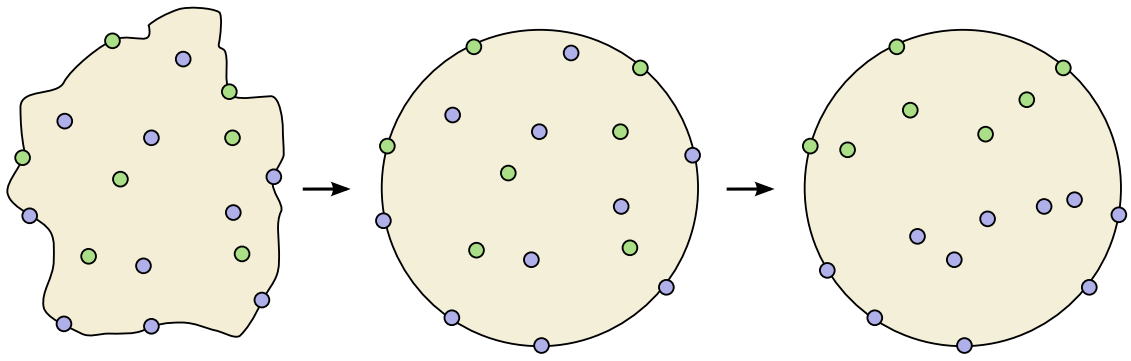
Think topologically



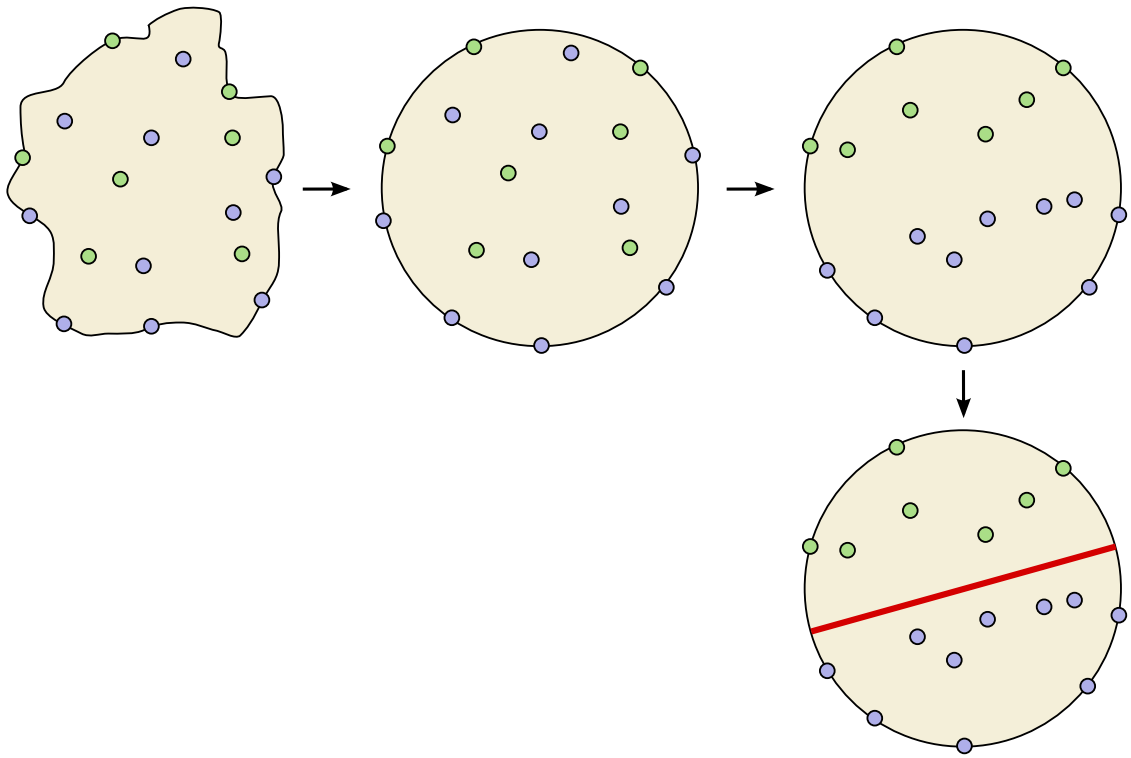
Think topologically



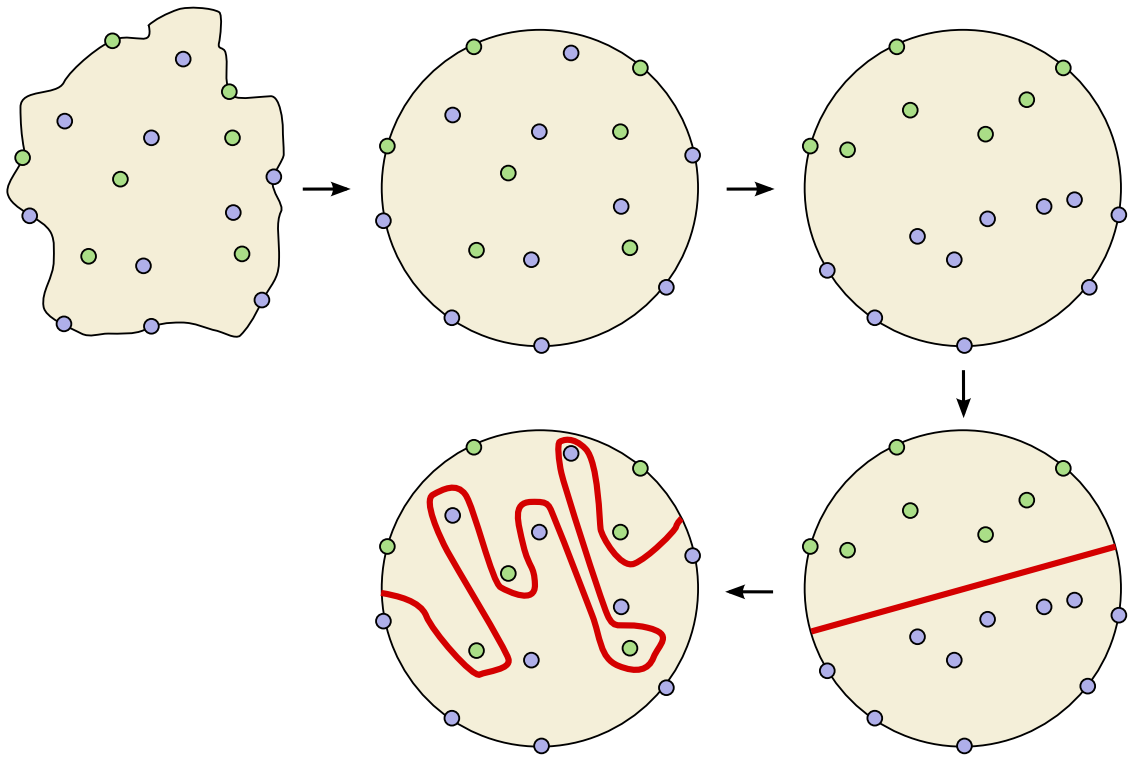
Think topologically



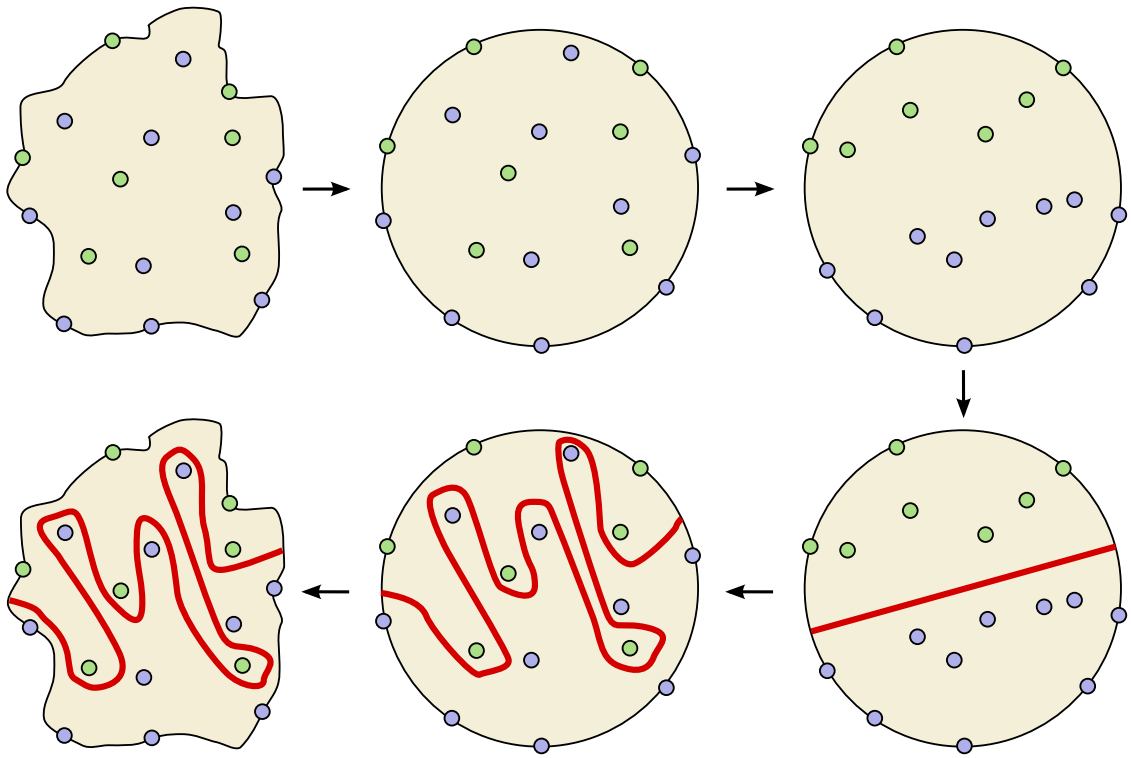
Think topologically



Think topologically



Think topologically



Dividing walls theorem

Dividing walls theorem

Theorem.

Dividing walls theorem

Theorem. A collection of blue and green towns

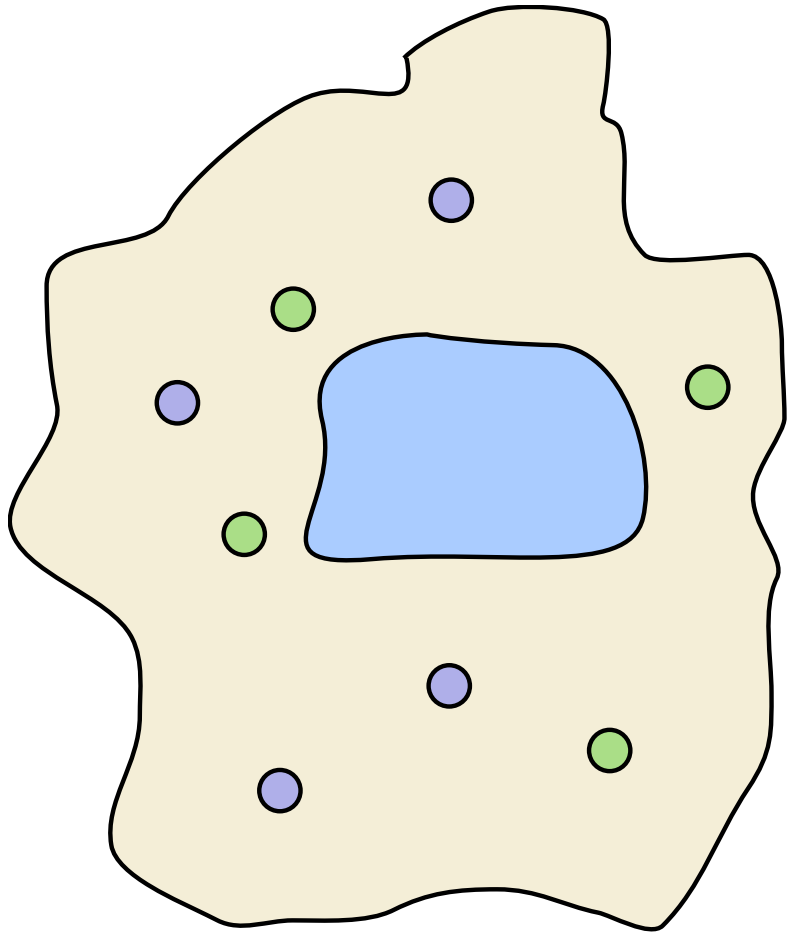
Dividing walls theorem

Theorem. A collection of blue and green towns can be separated (blue from green) by a dividing wall

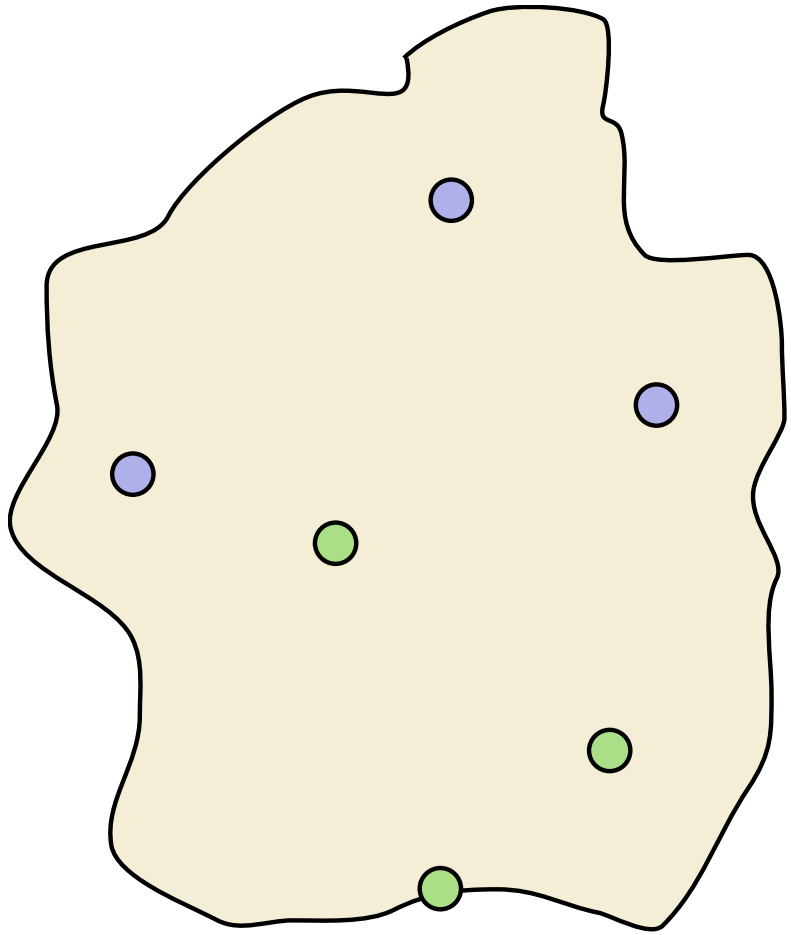
Dividing walls theorem

Theorem. A collection of blue and green towns can be separated (blue from green) by a dividing wall if and only if there are not four coastal towns configured in the same way as the simplist problematic example.

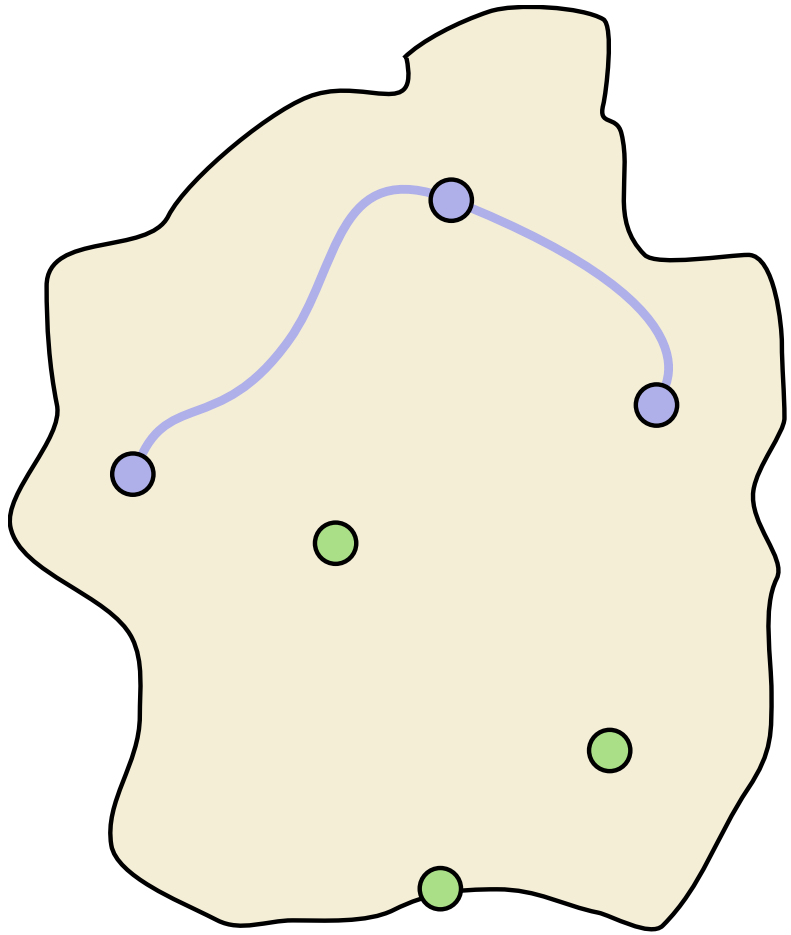
Lakes



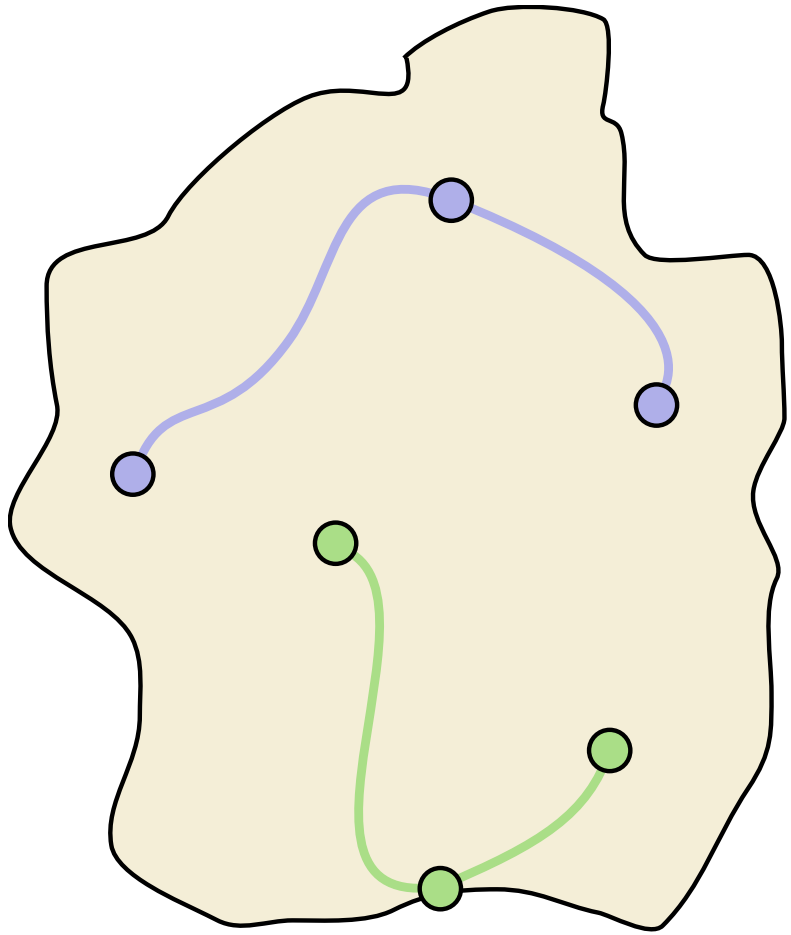
Connecting roads



Connecting roads



Connecting roads



Investigate dividing walls and connecting roads for three town colours.