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AN ENUMERATION OF S-CYCLIC SQS(26)

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1 Introduction.

A Steiner quadruple system of order v, denoted by SQS(v), is a pair (S, B), where S is a v-element set and B is a family of four-element subsets of S, usually called blocks, such that each three-element subset of S is contained in exactly one block. Such a system is said to be cyclic if B has an automorphism of order v. Any cyclic SQS(v) will be isomorphic to a system in which $S = V$, the set of residue classes modulo v, and B is invariant under the cyclic group $C_v = \langle i \to i+1 \pmod{v} \rangle$. A system of this type may be constructed as the union of orbits generated by the action of C_v, on four-element subsets of V. In this representation an orbit is said to be symmetric if it is invariant under the mapping $i \to -i \pmod{v}$ and an SQS(v) to be S-cyclic if it is isomorphic to a cyclic system all of whose orbits are symmetric.

A recent survey of Steiner quadruple systems is that by Lindner and Rosa [7]. They note that there are no cyclic SQS(v) for v = 8, 14, and 16 and just one cyclic SQS(10) which is also S-cyclic ([1], [5]). For $v = 20$ Phelps [8] has made a complete enumeration of cyclic systems. There are 29 non-isomorphic cyclic SQS(20) including one which is also S-cyclic [6]. Further, Diener [2] has shown that there are 21 non-isomorphic cyclic SQS(22), none of which are S-cyclic.

The next case to consider is $v = 26$. It appears that the number of non-isomorphic cyclic SQS(26) may be large but it is possible to enumerate the S-cyclic systems. Our method uses some of the results of a previous paper [4]. We prove that there are exactly 18 non-isomorphic S-cyclic SQS(26) of which most, but not all, are new.

2 General Results.

Let D be an orbit of blocks of V under C_v. We make the following definitions.

Definitions.

(a) D is said to be a *full orbit* or a *half orbit* according as to whether it contains v or $v/2$ distinct blocks. It is clear that these are the only possibilities for $v = 26$.

(b) D is said to be *suitable* if for every pair of distinct blocks $S, T \in D$ we have $|S \cap T| < 3$.

(c) Two distinct orbits D_1 and D_2, are said to be *compatible* if for every pair of blocks $S \in D_1$, and $T \in D_2$, we have $|S \cap T| < 3$.

It is easily seen (c.f. [8], [4]) that no S-cyclic SQS(v) may contain a half orbit. Hence any S-cyclic SQS(26) will consist of a union of pair-wise compatible, symmetric, suitable, full orbits (SSFOs).

In [4] we prove the following theorem.

THEOREM. If D is an SSFO not contained in a particular S-cyclic SQS(v) then there exist exactly two distinct full orbits D_1, and D_2, contained in the SQS(v) which are incompatible with D.

So if

$S =$ the total number of SSFOs,

$N =$ the number of SSFOs contained in the SQS(v),

$s_i =$ the total number of SSFOs which are incompatible with precisely i other SSFOs,

$n_i =$ the number of SSFOs contained in the SQS(v) which are incompatible with precisely i other SSFOs,

then clearly $\sum n_i = N$, $\sum i s_i = S$, and $n_i \leq s_i$. In addition, from the theorem, $\sum i n_i = 2(S - N)$ and $n_0 = s_0$, $n_1 = s_1$.

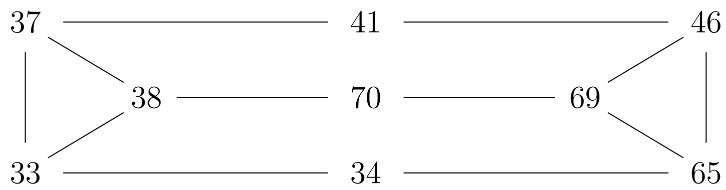
3 Enumeration of S-cyclic SQS(26).

A listing of details of all the symmetric orbits of blocks under C_{26} is given in Appendix 1. This gives $s_0 = 0$, $s_1 = 6$, $s_2 = 3$, $s_3 = 24$, $s_4 = 27$, $S = 60$, and $N = 25$.

We now proceed as follows:

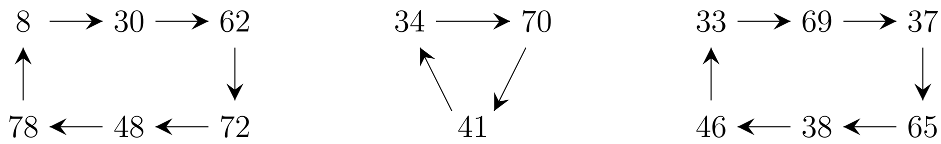
(a) $n_0 = 0$ and $n_1 = 6$. Hence orbits numbered 8, 30, 48, 62, 72, and 78 must be included in any S-cyclic SQS(26).

(b) Let $G = (X, E)$ be the graph whose vertex set X is the set of SSFOs and whose edge set E is the set of unordered pairs of X which are incompatible. The construction of an S-cyclic SQS(26), if it exists, is clearly equivalent to finding a maximum independent set of G. Now the graph G is disconnected and one of its components is



Note that this component contains all three orbits which contribute to s_2. There are three maximum independent sets of this component, namely orbits numbered 34, 41, 38 and 69; or 34, 70, 37 and 46; or 41, 70, 33 and 65.

(c) The integer 11 is a primitive root of 26 and the function $z \to 11z$ maps orbits to orbits. Its action on the orbits considered above is



Hence if we consider the following collections of orbits numbered:

(A1) 8, 30, 48, 62, 72, 78, 34, 41, 38 and 69,
(A2) 8, 30, 48, 62, 72, 78, 34, 70, 37 and 46,
(A3) 8, 30, 48, 62, 72, 78, 41, 70, 33 and 65,

these are isomorphic and also stabilized by the mapping $z \to 5z$ because $11^3 = 5$. Without loss of generality any of the three collections can be considered as necessarily included in any S-cyclic SQS(26). Hence $n_2 = 2$. Solving the equations $n_1 + n_2 + n_3 + n_4 = 25$ and $n_1 + 2n_2 + 3n_3 + 4n_4 = 2(60 - 25) = 70$ given by the theorem with the values deduced above gives $n_3 = 8$ and $n_4 = 9$.

(d) Consider the 24 orbits which contribute to s_3. Those numbered 18, 27, 43, 52, 55 and 63 are already excluded from any S-cyclic SQS(26) by the necessary inclusion of the orbits in (a) above. A further 6 of the 24 lie in the component of G described in (b) and we have shown that precisely 2 of these 6 are contained in any S-cyclic SQS(26). This leaves 12 of the 24 orbits and these partition into 6 mutually incompatible pairs, namely numbers 9 and 10, 19 and 21, 26 and 73, 36 and 79, 50 and 68, 53 and 64. Clearly one orbit from each pair must be included in any S-cyclic SQS(26).

(e) To complete the enumeration a computer search program was constructed. We find that there are 29 distinct collections of orbits each of which together with (A1), (A2), or (A3) form an S-cyclic SQS(26), thus giving a grand total of 87 realisations of such systems. Of these 29 distinct collections, 7 are stabilized by the mapping $z \to 5z$ and the other 22 form 11 pairs, the two collections of orbits in each pair being mapped to one another by the same mapping. Hence there exist at most 18 non-isomorphic classes of S-cyclic SQS(26). We list the details in Appendix 2, (X1) to (X7) being the stabilized collections and (Y1), (Y1') to (Y11), (Y11') the 11 pairs.

(f) It remains to prove that these 18 classes are indeed non-isomorphic. This can be done as follows. For any given S-cyclic SQS(26) the transitivity of the automorphism group implies that all the derived STS(25) are isomorphic. We select a derived STS(25) from each of the 18 systems A1X1, ..., A1X7, A1Y1, ..., A1Y11 and investigate its quadrilateral content. (A quadrilateral consists of 4 three-element sets of an STS(v) whose union has cardinality 6.) Although some of the derived systems contain the same number of quadrilaterals, more detailed inspection of the number of occurrences of elements and sets in the quadrilaterals proves that the systems are all non-isomorphic. Hence there are exactly 18 non-isomorphic S-cyclic SQS(26). The systems A1X7, A2Y7' (although this contains a misprint; 1.4.1.20 should read 4.1.4.17), and A3Y8 are given in [3] and A2X5 in [5]. All of the others appear to be new. For ease of reference, generating blocks for the collections of orbits (A1), (X1) to (X7), and (Y1) to (Y11) are listed in Appendix 3 in order that representations of the 18 non-isomorphic S-cyclic SQS(26) can be immediately obtained.

APPENDIX 1

Details of symmetric orbits of blocks under C_{26}. Orbits numbered 1 to 6 are half orbits. The remainder are full orbits.

Orbit number	Generating block	Unsuitable orbit or Incompatible orbits	Orbit mapped to under $z \to 11z$	Orbit mapped to under $z \to 5z$
1	$0, 1, 13, 14$	$8, 27$	2	5
2	$0, 2, 13, 15$	$30, 43$	4	3
3	$0, 3, 13, 16$	$48, 55$	6	2
4	$0, 4, 13, 17$	$62, 63$	5	6
5	$0, 5, 13, 18$	$52, 72$	3	1
6	$0, 6, 13, 19$	$18, 78$	1	4
7	$0, 1, 2, 3$	UNSUITABLE	67	71
8	$0, 1, 2, 14$	1(HALF ORBIT), 27	30	72
9	$0, 1, 3, 4$	$10, 11, 12$	68	73
10	$0, 1, 3, 24$	$9, 31, 32$	50	26
11	$0, 1, 4, 5$	$9, 12, 13, 14$	58	16
12	$0, 1, 4, 23$	$9, 11, 49, 50$	20	74
13	$0, 1, 5, 6$	$11, 14, 15, 16$	57	Stabilized
14	$0, 1, 5, 22$	$11, 13, 63, 64$	51	15
15	$0, 1, 6, 7$	$13, 16, 17, 18$	28	14
16	$0, 1, 6, 21$	$13, 15, 73, 74$	59	11
17	$0, 1, 7, 8$	$15, 18, 19, 20$	25	64
18	$0, 1, 7, 20$	6(HALF ORBIT), 15, 17, 78	27	63
19	$0, 1, 8, 9$	$17, 20, 21$	26	75
20	$0, 1, 8, 19$	$17, 19, 58, 68$	23	60
21	$0, 1, 9, 10$	$19, 23, 24$	73	36
22	$0, 1, 9, 18$	UNSUITABLE	71	76
23	$0, 1, 10, 11$	$21, 24, 25, 26$	74	31
24	$0, 1, 10, 17$	$21, 23, 40, 56$	16	35
25	$0, 1, 11, 12$	$23, 26, 27, 28$	44	32
26	$0, 1, 11, 16$	$23, 25, 73$	54	10
27	$0, 1, 12, 13$	1(HALF ORBIT), 8, 25, 28	43	52
28	$0, 1, 12, 15$	$25, 27, 57, 59$	39	51
29	$0, 2, 4, 6$	UNSUITABLE	61	66
30	$0, 2, 4, 15$	2(HALF ORBIT), 43	62	48
31	$0, 2, 5, 7$	$10, 32, 35, 36$	12	23
32	$0, 2, 5, 23$	$10, 31, 51, 52$	49	25
33	$0, 2, 6, 8$	$34, 37, 38$	69	65
34	$0, 2, 6, 22$	$33, 65$	70	Stabilized
35	$0, 2, 7, 9$	$31, 36, 39, 40$	11	24
36	$0, 2, 7, 21$	$31, 35, 75$	9	21
31	$0, 2, 8, 10$	$33, 38, 41$	65	46
38	$0, 2, 8, 20$	$33, 37, 70$	46	69
39	$0, 2, 9, 11$	$35, 40, 43, 44$	14	56
40	$0, 2, 9, 19$	$24, 35, 39, 56$	13	Stabilized

Orbit number	Generating block	Unsuitable orbit or Incompatible orbits	Orbit mapped to under $z \to 11z$	Orbit mapped to under $z \to 5z$
41	$0, 2, 10, 12$	$37, 46$	34	Stabilized
42	$0, 2, 10, 18$	UNSUITABLE	66	45
43	$0, 2, 11, 13$	2(HALF ORBIT), $30, 39, 44$	63	55
44	$0, 2, 11, 17$	$39, 43, 54, 74$	64	49
45	$0, 2, 12, 14$	UNSUITABLE	29	42
46	$0, 2, 12, 16$	$41, 65, 69$	33	37
47	$0, 3, 6, 9$	UNSUITABLE	76	67
48	$0, 3, 6, 16$	3(HALF ORBIT), 55	78	30
49	$0, 3, 7, 10$	$12, 50, 55, 56$	17	44
50	$0, 3, 7, 22$	$12, 49, 68$	19	54
51	$0, 3, 8, 11$	$32, 52, 57, 58$	56	28
52	$0, 3, 8, 21$	5(HALF ORBIT), $32, 51, 72$	55	27
53	$0, 3, 9, 12$	$54, 59, 60$	36	68
54	$0, 3, 9, 20$	$44, 53, 74$	75	50
55	$0, 3, 10, 13$	3(HALF ORBIT), $48, 49, 56$	18	43
56	$0, 3, 10, 19$	$24, 40, 49, 55$	15	39
57	$0, 3, 11, 14$	$28, 51, 58, 59$	40	Stabilized
58	$0, 3, 11, 18$	$20, 51, 57, 68$	24	59
59	$0, 3, 12, 15$	$28, 53, 57, 60$	35	58
60	$0, 3, 12, 17$	$53, 59, 64, 75$	31	20
61	$0, 4, 8, 12$	UNSUITABLE	42	77
62	$0, 4, 8, 17$	4(HALF ORBIT), 63	72	78
63	$0, 4, 9, 13$	4(HALF ORBIT), $14, 62, 64$	52	18
64	$0, 4, 9, 21$	$14, 60, 63, 75$	32	17
65	$0, 4, 10, 14$	$34, 46, 69$	38	33
66	$0, 4, 10, 20$	UNSUITABLE	77	29
67	$0, 4, 11, 15$	UNSUITABLE	22	47
68	$0, 4, 11, 19$	$20, 50, 58$	21	53
69	$0, 4, 12, 16$	$46, 65, 70$	37	38
70	$0, 4, 12, 18$	$38, 69$	41	Stabilized
71	$0, 5, 10, 15$	UNSUITABLE	47	7
72	$0, 5, 10, 18$	5(HALF ORBIT), 52	48	8
73	$0, 5, 11, 16$	$16, 26, 74$	53	9
74	$0, 5, 11, 20$	$16, 44, 54, 73$	60	12
75	$0, 5, 12, 17$	$36, 60, 64$	10	19
76	$0, 5, 12, 19$	UNSUITABLE	7	22
77	$0, 6, 12, 18$	UNSUITABLE	45	61
78	$0, 6, 12, 19$	6(HALF ORBIT), 18	8	62

APPENDIX 2

Collections of orbits which with (A1), (A2) or (A3) form an S-cyclic SQS(26). (X1) to (X7) are stabilized by the mapping $z \to 5z$. (Yn) and (Yn'), $n = 1, 2, \ldots, 11$ are mapped to each other by the same mapping.

	6 orbits contributing to n_3	9 orbits contributing to n_4
(X1)	9,19,73,75,68,53	14,15,23,28,31,40,44,49,51
(X2)	9,19,73,75,68,53	14,15,24,25,32,35,44,49,57
(X3)	9,21,73,36,50,54	13,17,25,32,39,56,58,59,64
(X4)	9,21,73,36,50,54	14,15,20,25,32,39,56,57,60
(X5)	9,21,73,36,68 53	13,17,25,32,40,44,49,57,64
(X6)	10,21,26,36,68,53	11,16,17,28,40,44,49,51,64
(X7)	10,21,26,36,68,53	12,13,17,28,39,51,56,64,74
(Y1)	9,19,26,75,68,53	14,15,24,28,31,39,49,51,74
(Y1')	73,75,10,19,53,68	15,14,35,51,23,56,44,28,12
(Y2)	9,19,73,36,50,54	14,15,23,28,32,39,56,58,60
(Y2')	73,75, 9,21,54,50	15,14,31,51,25,56,39,59,20
(Y3)	9,19,73,36,68,54	14,15,24,25,32,39,49,57,60
(Y3')	73,75, 9,21,53,50	15,14,35,32,25,56,44,57,20
(Y4)	9,19,73,75,50,53	14,15,23,28,32,35,44,56,58
(Y4')	73,75, 9,19,54,68	15,14,31,51,25,24,49,39,59
(Y5)	9,21,26,36,50,53	13,17,28,32,39,56,58,64,74
(Y5')	73,36,10,21,54,68	13,64,51,25,56,39,59,17,12
(Y6)	9,21,26,36,50,54	14,16,17,28,32,39,56,58,60
(Y6')	73,36,10,21,54,50	15,11,64,51,25,56,39,59,20
(Y7)	9,21,26,75,50,53	14,15,20,28,31,39,51,56,74
(Y7')	73,36,10,19,54,68	15,14,60,51,23,56,28,39,12
(Y8)	9,21,26,75,50,53	14,16,17,28,32,35,44,56,58
(Y8')	73,36,10,19,54,68	15,11,64,51,25,24,49,39,59
(Y9)	9,21,26,75,68,53	14,16,17,28,31,40,44,49,51
(Y9')	73,36,10,19,53,68	15,11,64,51,23,40,49,44,28
(Y10)	10,19,26,36,68,53	11,15,24,28,39,49,51,64,74
(Y10')	26,75,10,21,53,68	16,14,35,51,56,44,28,17,12
(Y11)	10,21,26,36,50,53	11,15,20,28,39,51,56,64,74
(Y11')	26,36,10,21,54,68	16,14,60,51,56,28,39,17,12

APPENDIX 3

List of generating blocks for the 18 non-isomorphic S-cyclic SQS(26). These are obtained by taking collection (A1) with each of the collections (X1) to (X7), and (Y1) to (Y11). Systems A1Xn, $n = 1, 2, \ldots, 7$ have 5 (and 21) as additional multiplier automorphisms.

(A1)

$\{0,1,2,14\}$, $\{0,2,4,15\}$, $\{0,3,6,16\}$, $\{0,4,8,17\}$, $\{0,5,10,18\}$,
$\{0,6,12,19\}$, $\{0,2,6,22\}$, $\{0,2,10,12\}$, $\{0,2,8,20\}$, $\{0,4,12,16\}$.

(X1)

$\{0,1,3,4\}$, $\{0,1,8,9\}$, $\{0,5,11,16\}$, $\{0,5,12,17\}$, $\{0,4,11,19\}$,
$\{0,3,9,12\}$, $\{0,1,5,22\}$, $\{0,1,6,7\}$, $\{0,1,10,11\}$, $\{0,1,12,15\}$,
$\{0,2,5,7\}$, $\{0,2,9,19\}$, $\{0,2,11,17\}$, $\{0,3,7,10\}$, $\{0,3,8,11\}$.

(X2)

$\{0,1,3,4\}$, $\{0,1,8,9\}$, $\{0,5,11,16\}$, $\{0,5,12,17\}$, $\{0,4,11,19\}$,
$\{0,3,9,12\}$, $\{0,1,5,22\}$, $\{0,1,6,7\}$, $\{0,1,10,17\}$, $\{0,1,11,12\}$,
$\{0,2,5,23\}$, $\{0,2,7,9\}$, $\{0,2,11,17\}$, $\{0,3,7,10\}$, $\{0,3,11,14\}$.

(X3)

$\{0,1,3,4\}$, $\{0,1,9,10\}$, $\{0,5,11,16\}$, $\{0,2,7,21\}$, $\{0,3,7,22\}$,
$\{0,3,9,20\}$, $\{0,1,5,6\}$, $\{0,1,7,8\}$, $\{0,1,11,12\}$, $\{0,2,5,23\}$,
$\{0,2,9,11\}$, $\{0,3,10,19\}$, $\{0,3,11,18\}$, $\{0,3,12,15\}$, $\{0,4,9,21\}$.

(X4)

$\{0,1,3,4\}$, $\{0,1,9,10\}$, $\{0,5,11,16\}$, $\{0,2,7,21)$, $\{0,3,7,22\}$,
$\{0,3,9,20\}$, $(0,1,5,22)$, $\{0,1,6,7\}$, $\{0,1,8,19\}$, $\{0,1,11,12\}$,
$\{0,2,5,23\}$, $\{0,2,9,11\}$, $\{0,3,10,19\}$, $\{0,3,11,14\}$, $\{0,3,12,17\}$.

(X5)

$\{0,1,3,4\}$, $\{0,1,9,10\}$, $\{0,5,11,16\}$, $\{0,2,7,21\}$, $\{0,4,11,19\}$,
$\{0,3,9,12\}$, $\{0,1,5,6\}$, $\{0,1,7,8\}$, $\{0,1,11,12\}$, $\{0,2,5,23\}$,
$\{0,2,9,19\}$, $\{0,2,11,17\}$, $\{0,3,7,10\}$, $\{0,3,11,14\}$, $\{0,4,9,21\}$.

(X6)

$\{0,1,3,24\}$,	$\{0,1,9,10\}$,	$\{0,1,11,16\}$,	$\{0,2,7,21\}$,	$\{0,4,11,19\}$,
$\{0,3,9,12\}$,	$\{0,1,4,5\}$,	$\{0,1,6,21\}$,	$\{0,1,7,8\}$,	$\{0,1,12,15\}$,
$\{0,2,9,19\}$,	$\{0,2,11,17\}$,	$\{0,3,7,10\}$,	$\{0,3,8,11\}$,	$\{0,4,9,21\}$.

(X7)

$\{0,1,3,24\}$,	$\{0,1,9,10\}$,	$\{0,1,11,16\}$,	$\{0,2,7,21\}$,	$\{0,4,11,19\}$,
$\{0,3,9,12\}$,	$\{0,1,4,23\}$,	$\{0,1,5,6\}$,	$\{0,1,7,8\}$,	$\{0,1,12,15\}$,
$\{0,2,9,11\}$,	$\{0,3,8,11\}$,	$\{0,3,10,19\}$,	$\{0,4,9,21\}$,	$\{0,5,11,20\}$.

(Y1)

$\{0,1,3,4\}$,	$\{0,1,8,9\}$,	$\{0,1,11,16\}$,	$\{0,5,12,17\}$,	$\{0,4,11,19\}$,
$\{0,3,9,12\}$,	$\{0,1,5,22\}$,	$\{0,1,6,7\}$,	$\{0,1,10,17\}$,	$\{0,1,12,15\}$,
$\{0,2,5,7\}$,	$\{0,2,9,11\}$,	$\{0,3,7,10\}$,	$\{0,3,8,11\}$,	$\{0,5,11,20\}$.

(Y2)

$\{0,1,,3,4\}$,	$\{0,1,8,9\}$,	$\{0,5,11,16\}$,	$\{0,2,7,21\}$,	$\{0,3,7,22\}$,
$\{0,3,9,20\}$,	$\{0,1,5,22\}$,	$\{0,1,6,7\}$,	$\{0,1,10,11\}$,	$\{0,1,12,15\}$,
$\{0,2,5,23\}$,	$\{0,2,9,11\}$,	$\{0,3,10,19\}$,	$\{0,3,11,18\}$,	$\{0,3,12,17\}$.

(Y3)

$\{0,1,3,4\}$,	$\{0,1,8,9\}$,	$\{0,5,11,16\}$,	$\{0,2,7,21\}$,	$\{0,4,11,19\}$,
$\{0,3,9,20\}$,	$\{0,1,5,22\}$,	$\{0,1,6,7\}$,	$\{0,1,10,17\}$,	$\{0,1,11,12\}$,
$\{0,2,5,23\}$,	$\{0,2,9,11\}$,	$\{0,3,7,10\}$,	$\{0,3,11,14\}$,	$\{0,3,12,17\}$.

(Y4)

$\{0,1,3,4\}$,	$\{0,1,8,9\}$,	$\{0,5,11,16\}$,	$\{0,5,12,17\}$,	$\{0,3,7,22\}$,
$\{0,3,9,12\}$,	$\{0,1,5,22\}$,	$\{0,1,6,7\}$,	$\{0,1,10,11\}$,	$\{0,1,12,15\}$,
$\{0,2,5,23\}$,	$\{0,2,7,9\}$,	$\{0,2,11,17\}$,	$\{0,3,10,19\}$,	$\{0,3,11,18\}$.

(Y5)

$\{0,1,3,4\}$,	$\{0,1,9,10\}$,	$\{0,1,11,16\}$,	$\{0,2,7,21\}$,	$\{0,3,7,22\}$,
$\{0,3,9,12\}$,	$\{0,1,5,6\}$,	$\{0,1,7,8\}$,	$\{0,1,12,15\}$,	$\{0,2,5,23\}$,
$\{0,2,9,11\}$,	$\{0,3,10,19\}$,	$\{0,3,11,18\}$,	$\{0,4,9,21\}$,	$\{0,5,11,20\}$.

(Y6)

$$\begin{array}{lllll}
\{0,1,3,4\}, & \{0,1,9,10\}, & \{0,1,11,16\}, & \{0,2,7,21\}, & \{0,3,7,22\}, \\
\{0,3,9,20\}, & \{0,1,5,22\}, & \{0,1,6,21\}, & \{0,1,7,8\}, & \{0,1,12,15\}, \\
\{0,2,5,23\}, & \{0,2,9,11\}, & \{0,3,10,19\}, & \{0,3,11,18\}, & \{0,3,12,17\}.
\end{array}$$

(Y7)

$$\begin{array}{lllll}
\{0,1,3,4\}, & \{0,1,9,10\}, & \{0,1,11,16\}, & \{0,5,12,17\}, & \{0,3,7,22\}, \\
\{0,3,9,12\}, & \{0,1,5,22\}, & \{0,1,6,7\}, & \{0,1,8,19\}, & \{0,1,12,15\}, \\
\{0,2,5,7\}, & \{0,2,9,11\}, & \{0,3,8,11\}, & \{0,3,10,19\}, & \{0,5,11,20\}.
\end{array}$$

(Y8)

$$\begin{array}{lllll}
\{0,1,3,4\}, & \{0,1,9,10\}, & \{0,1,11,16\}, & \{0,5,12,17\}, & \{0,3,7,22\}, \\
\{0,3,9,12\}, & \{0,1,5,22\}, & \{0,1,6,21\}, & \{0,1,7,8\}, & \{0,1,12,15\}, \\
\{0,2,5,23\}, & \{0,2,7,9\}, & \{0,2,11,17\}, & \{0,3,10,19\}, & \{0,3,11,18\}.
\end{array}$$

(Y9)

$$\begin{array}{lllll}
\{0,1,3,4\}, & \{0,1,9,10\}, & \{0,1,11,16\}, & \{0,5,12,17\}, & \{0,4,11,19\}, \\
\{0,3,9,12\}, & \{0,1,5,22\}, & \{0,1,6,21\}, & \{0,1,7,8\}, & \{0,1,12,15\}, \\
\{0,2,5,7\}, & \{0,2,9,19\}, & \{0,2,11,17\}, & \{0,3,7,10\}, & \{0,3,8,11\}.
\end{array}$$

(Y10)

$$\begin{array}{lllll}
\{0,1,3,24\}, & \{0,1,8,9\}, & \{0,1,11,16\}, & \{0,2,7,21\}, & \{0,4,11,19\}, \\
\{0,3,9,12\}, & \{0,1,4,5\}, & \{0,1,6,7\}, & \{0,1,10,17\}, & \{0,1,12,15\}, \\
\{0,2,9,11\}, & \{0,3,7,10\}, & \{0,3,8,11\}, & \{0,4,9,21\}, & \{0,5,11,20\}.
\end{array}$$

(Y11)

$$\begin{array}{lllll}
\{0,1,3,24\}, & \{0,1,9,10\}, & \{0,1,11,16\}, & \{0,2,7,21\}, & \{0,3,7,22\}, \\
\{0,3,9,12\}, & \{0,1,4,5\}, & \{0,1,6,7\}, & \{0,1,8,19\}, & \{0,1,12,15\}, \\
\{0,2,9,11\}, & \{0,3,8,11\}, & \{0,3,10,19\}, & \{0,4,9,21\}, & \{0,5,11,20\}.
\end{array}$$

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