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Derived Steiner Triple Systems of Order 15

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1 Introduction

Denote a Steiner system by $\mathrm{S}(t, k, v)$ where the parameters have their usual meaning. It is an elementary proposition that if any point of a Steiner system is chosen, all blocks not containing the point are deleted, and the point itself is then deleted from all of the remaining blocks, what remains is another Steiner system $\mathrm{S}(t-1, k-1, v-1)$. The latter system is said to be derived from the former. It is well known that necessary and sufficient conditions are as follows: for a Steiner triple system $\mathrm{S}(2, 3, v)$ or $\mathrm{STS}(v)$, $v \equiv 1$ or $3 \pmod 6$ while for a Steiner quadruple system $\mathrm{S}(3, 4, v)$ or $\mathrm{SQS}(v)$, $v \equiv 2$ or $4 \pmod 6$. Such v are called admissible. It follows that there exists a derived Steiner triple system for every admissible order. However, whether or not every Steiner triple system is derived is a fascinating open question.

For $v = 7$ and 9, the Steiner triple systems are unique up to a isomorphism and are therefore derived. The case when $v = 13$ was solved by Mendelsohn and Hung [7] who showed that both of the two non-isomorphic systems which exist for this order are also derived. There are 80 non-isomorphic Steiner triple systems of order 15 (see [2] and [4]). In this paper we shall use the listing of these given by Bussemaker and Seidel [1], and also given in [5] where it is probably more easily accessible. The present state of knowledge concerning the derivability of these systems is given in the survey paper by Phelps [10]. It rests heavily on general theorems, also by Phelps, in earlier papers [8], [9]. In the first of these he proves:

Theorem A (Phelps [8])
A Steiner triple system of order $2v+1$ with a derived Steiner triple system of order v is itself derived.

This theorem shows that 23 of the 80 systems, namely #1-22 and 61, are derived since they contain the STS(7), $\{1,2,3\},\{1,4,5\},\{1,6,7\},\{2,4,6\},\{2,5,7\},\{3,4,7\},\{3,5,6\}$. In the second paper a theorem equivalent to the following is proved:

Theorem B (Phelps [9])
If a Steiner triple system of order $2v+1$ contains all but one of the blocks of a Steiner triple system of order v, and this STS(v) is derived then the STS($2v+1$) is also derived. (In [2] an STS(v) with one block missing is called a semi-head).

This theorem shows that 15 more systems, namely #23-34, 62, 63 and 64 are derived since they contain the semi-head $\{1,2,3\},\{1,4,5\},\{1,6,7\},\{2,4,6\},\{2,5,7\},\{3,4,7\}$.

Finally in [10], Phelps states that he has himself determined that #35 and 53 are derived and that Gibbons [3] has added #59, 70 and 76. The SQS(16)s containing these STS(15)s as derived systems are exhibited in the recent encyclopaedic paper by Mathon, Phelps and Rosa [6]. Thus the total number of known derived STS(15)s is 43. In this paper we raise this number to 66.

2 Methodology

Our general methodology is an extension of that used by Phelps [8], [9], in the proof of his theorems quoted above. Our method is applicable only to Steiner triple systems of order 15 and involves the use of a computer search. We analyse the situation in which an STS(15) contains an STS(7) apart from two blocks (a demi-semi-head?). First we need a definition.

Definition. A quadrilateral consists of four blocks of a Steiner triple system whose union has cardinality six.

It is clear that a quadrilateral must have the following configuration: $\{a,b,c\},\{a,y,z\},\{x,b,z\},\{x,y,c\}$. When such a collection appears in a Steiner triple system it may be removed and replaced by the "opposite" quadrilateral $\{x,y,z\},\{x,b,c\},\{a,y,c\},\{a,b,z\}$ to form a different (but possibly isomorphic) Steiner triple system. Gibbons [3] has shown that precisely 79 of the 80 STS(15)s contain at least one quadrilateral and that these may be transformed into one another by repeated changing of quadrilaterals as described.

Note firstly that the inclusion of a quadrilateral within an STS(15) is equivalent to the STS(15) containing five of the seven blocks of an STS(7). We now proceed with the analysis.

Let the quadrilateral be $\{a_1, a_3, a_5\}, \{a_1, a_4, a_6\}, \{a_2, a_3, a_6\}, \{a_2, a_4, a_5\}$. Identify the three pairs of elements which are not included in the quadrilateral and list the three blocks of the STS(15) which contain these pairs. Suppose these are $\{a_1, a_2, x\}, \{a_3, a_4, y\}, \{a_5, a_6, z\}$. Then none of x, y and z can equal any a_i and, moreover, we can assume that x, y and z are themselves unequal (for otherwise the STS(15) would contain either an STS(7) or a semi-head which can be dealt with by Phelps' theorems). Select one of these latter three blocks. Without loss of generality we will choose $\{a_1, a_2, x\}$. Next identify the blocks which contain the pairs $\{a_3, x\}, \{a_4, x\}, \{a_5, x\}, \{a_6, x\}$. Let these be $\{a_3, x, b_3\}, \{a_4, x, b_4\}, \{a_5, x, b_5\}, \{a_6, x, b_6\}$ The b_i s must be distinct from one another and from each of the a_i s. Also, $y \neq b_3$ or b_4 and $z \neq b_5$ or b_6. However, it is possible for y to be equal to b_5 or b_6, or z to be equal to b_3 or b_4, (but not simultaneously). The above blocks are 11 of the 35 blocks in an STS(15).

Since each element occurs 7 times within an STS(15), there are in addition four more blocks containing a_1 and likewise for a_2, three more blocks containing a_3 and likewise for a_4, a_5 and a_6, and two more blocks containing x, all these blocks being distinct and numbering 22 in all. It is left to identify the remaining two blocks. A counting argument shows that these contain the 'six' elements y, z, b_3, b_4, b_5, b_6. It is to be understood that if, for example, $y = b_5$ then this element appears twice, that is once in each of the two blocks. The exact partition of the elements into the two blocks is not determined. We now make the further assumption that these two blocks are $\{b_3, b_4, y\}$ and $\{b_5, b_6, z\}$ i.e. that the configuration of the STS(15) is as given below.

$$\begin{array}{llll}
\{a_1, a_3, a_5\}, & \{a_1, a_4, a_6\}, & \{a_2, a_3, a_6\}, & \{a_2, a_4, a_5\}, \\
\{a_1, a_2, x\}, & \{a_3, a_4, y\}A, & \{a_5, a_6, z\}B, & \\
\{a_3, x, b_3\}A, & \{a_4, x, b_4\}A, & \{a_5, x, b_5\}B, & \{a_6, x, b_6\}B, \\
\{b_3, b_4, y\}A, & \{b_5, b_6, z\}B, & &
\end{array}$$

together with the 22 blocks identified above.

The four blocks labelled A form a quadrilateral as do the four labelled B. Replacing these with the "opposite" quadrilaterals gives the following transformed STS(15).

$$
\begin{array}{llll}
\{a_1,a_3,a_5\}, & \{a_1,a_4,a_6\}, & \{a_2,a_3,a_6\}, & \{a_2,a_4,a_5\}, \\
\{a_1,a_2,x\}, & \{a_3,a_4,x\}A, & \{a_5,a_6,x\}B, & \\
\{a_1, \quad\quad\}, & \{a_1, \quad\quad\}, & \{a_1, \quad\quad\}, & \{a_1, \quad\quad\}, \\
\{a_2, \quad\quad\}, & \{a_2, \quad\quad\}, & \{a_2, \quad\quad\}, & \{a_2, \quad\quad\}, \\
\{a_3,b_3,y\}A, & \{a_3, \quad\quad\}, & \{a_3, \quad\quad\}, & \{a_3, \quad\quad\}, \\
\{a_4,b_4,y\}A, & \{a_4, \quad\quad\}, & \{a_4, \quad\quad\}, & \{a_4, \quad\quad\}, \\
\{a_5,b_5,z\}B, & \{a_5, \quad\quad\}, & \{a_5, \quad\quad\}, & \{a_5, \quad\quad\}, \\
\{a_6,b_6,z\}B, & \{a_6, \quad\quad\}, & \{a_6, \quad\quad\}, & \{a_6, \quad\quad\}, \\
\{x,b_3,b_4\}A, & \{x,b_5,b_6\}B, & \{x, \quad\quad\}, & \{x, \quad\quad\}.
\end{array}
$$

This latter STS(15) contains an STS(7) on the elements $a_1, a_2, a_3, a_4, a_5, a_6$, and x and hence may be extended to an SQS(16) using Phelps' techniques. The method is as follows:

(1) The STS(7) is extended to an SQS(8) with one extra element, say ∞.

(2) The other 28 blocks of the STS(15) all have the element ∞ adjoined to them.

(3) Another SQS(8) is formed on the elements b_3, b_4, b_5, b_6, y, z, and two further elements b_1 and b_2.

(4) A one-factorization of a graph K_8 whose vertices are the elements $a_1, a_2, a_3, a_4, a_5, a_6, x$, and ∞ is formed. The system SQS(16) is then completed by taking each edge $\{a_i, a_j\}$, $i \neq j$, in turn and identifying the edge $\{\infty, a_k\}$ or $\{\infty, x\}$ within the same one-factor. The element a_k, or x occurs four times in blocks of the STS(15) with disjoint pairs of elements from the set $b_1, b_2, b_3, b_4, b_5, b_6, y, z$. Four new blocks are formed each containing one of these pairs together with $\{a_i, a_j\}$, $i \neq j$ or $\{a_i x\}$.

Clearly stages (3) and (4) contain some flexibility. In carrying out these steps, it may be possible to arrange that the four 3-blocks in each of the two quadrilaterals (A and B) of the original, untransformed STS(15) receive the same fourth element in the SQS(16). It is then possible to transform the SQS(16) to another SQS(16) containing the original STS(15) as a derived subsystem.

Our method was, therefore, to search each STS(15) for quadrilaterals and determine which of these extend to the configuration described. This we did by computer. Using the configuration to extend the STS(15) to an SQS(16) was then undertaken by hand and was found to be a not too onerous task.

3 Results

In searching for the configuration described in the previous section, the computer results indicated that in addition to the 15 systems identified in [9], 9 further STS(15)s, including both of the additional systems considered by Phelps [10] and one of the three considered by Gibbons [3] have derived semi-heads. Hence it follows from theorem B that these systems are derived.

The systems, together with their semi-heads, are:

#35,	$\{1,4,5\}$,	$\{1,10,11\}$,	$\{1,12,13\}$,	$\{4,11,13\}$,	$\{5,10,13\}$,	$\{5,11,12\}$
#39,	$\{1,6,7\}$,	$\{1,8,9\}$,	$\{6,8,15\}$,	$\{6,9,13\}$,	$\{7,8,13\}$,	$\{7,9,15\}$
#40,	$\{1,4,5\}$,	$\{1,10,11\}$,	$\{1,14,15\}$,	$\{4,10,15\}$,	$\{4,11,14\}$,	$\{5,10,14\}$
#41,	$\{1,6,7\}$,	$\{1,8,9\}$,	$\{6,8,15\}$,	$\{6,9,13\}$,	$\{7,8,13\}$,	$\{7,9,15\}$
#47,	$\{1,6,7\}$,	$\{1,8,9\}$,	$\{1,14,15\}$,	$\{6,8,14\}$,	$\{6,9,15\}$,	$\{7,9,14\}$
#53,	$\{2,4,6\}$,	$\{2,13,15\}$,	$\{4,10,15\}$,	$\{4,11,13\}$,	$\{6,10,13\}$,	$\{6,11,15\}$
#54,	$\{2,4,6\}$,	$\{2,9,11\}$,	$\{4,9,15\}$,	$\{4,11,14\}$,	$\{6,9,14\}$,	$\{6,11,15\}$
#58,	$\{1,10,11\}$,	$\{1,14,15\}$,	$\{4,10,15\}$,	$\{4,11,14\}$,	$\{6,10,14\}$,	$\{6,11,15\}$
#59,	$\{1,6,7\}$,	$\{1,8,9\}$,	$\{1,14,15\}$,	$\{6,9,15\}$,	$\{7,8,15\}$,	$\{7,9,14\}$

Apart from these, 19 STS(15)s (including the other two considered by Gibbons), contain the configuration described in the previous section. Using the configuration we could find in each case an SQS(16) with the STS(15) as a derived system. These SQS(16)s are given in the Appendix; the 35 blocks of each STS(15) all have a further element (16) adjoined to them and these blocks are listed down the first column. Thus the STS(15)s may be checked against the listings in [1] or [5] by the reader. The SQS(16)s have been checked by the authors using a computer checking program.

The situation concerning derived STS(15)s is now as follows:

1. 23 systems contain an STS(7) and are thus derived by theorem A. These are #1-22 and 61.

2. 24 systems contain a semi-head and are thus derived by theorem B. These are #23-35, 39, 40, 41, 47, 53, 54, 58, 59, 62, 63 and 64.

3. 19 systems contain the configuration described in this paper and are derived as indicated in the Appendix. These are #36, 38, 43-46, 48-52, 55, 56, 57, 60, 70, 74, 75 and 76.

4. 14 systems remain whose derivability is still undetermined. These are #37, 42, 65-69, 71, 72, 73 and 77-80.

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APPENDIX

SYSTEM NUMBER 36

1	2	3	16	2	6	13	14	1	4	13	14	3	7	11	15
1	4	5	16	2	7	11	12	2	6	8	11	2	4	5	13
1	6	7	16	2	3	12	13	3	8	11	12	4	5	7	12
1	8	9	16	2	3	6	7	1	8	11	13	3	4	5	6
1	10	11	16	1	2	7	13	2	6	9	15	2	8	9	13
1	12	13	16	1	2	6	12	3	9	12	15	7	8	9	12
1	14	15	16	6	7	12	13	1	9	13	15	3	6	8	9
2	4	6	16	1	3	7	12	5	7	10	11	2	10	11	13
2	5	7	16	1	3	6	13	2	5	6	10	3	6	10	11
2	8	10	16	4	8	9	15	3	5	10	12	7	12	14	15
2	9	11	16	8	10	11	15	1	5	10	13	3	6	14	15
2	12	14	16	9	10	14	15	2	4	7	9	4	6	8	13
2	13	15	16	4	5	10	15	3	4	9	13	2	4	8	12
3	4	8	16	5	8	14	15	1	4	6	9	1	4	7	8
3	5	9	16	5	9	11	15	3	5	11	13	5	6	9	13
3	6	12	16	5	8	9	10	1	5	6	11	2	5	9	12
3	7	13	16	4	5	9	14	2	7	8	15	1	5	7	9
3	10	15	16	4	5	8	11	3	8	13	15	6	10	13	15
3	11	14	16	8	9	11	14	1	6	8	15	2	10	12	15
4	7	14	16	4	9	10	11	10	11	12	14	1	7	10	15
4	9	12	16	4	8	10	14	2	7	10	14	1	7	11	14
4	10	13	16	2	3	5	15	3	10	13	14	6	7	8	10
4	11	15	16	1	5	12	15	1	6	10	14	1	3	8	10
5	6	15	16	5	7	13	15	1	2	4	10	8	10	12	13
5	8	13	16	2	3	8	14	4	6	10	12	6	7	9	11
5	10	14	16	1	8	12	14	3	4	7	10	1	3	9	11
5	11	12	16	7	8	13	14	1	2	5	8	9	11	12	13
6	8	14	16	2	3	9	10	5	6	8	12	2	4	14	15
6	9	10	16	1	9	10	12	3	5	7	8	4	6	7	15
6	11	13	16	7	9	10	13	1	2	9	14	1	3	4	15
7	8	11	16	4	6	11	14	6	9	12	14	4	12	13	15
7	9	15	16	2	3	4	11	3	7	9	14	2	5	11	14
7	10	12	16	1	4	11	12	11	13	14	15	5	6	7	14
8	12	15	16	4	7	11	13	1	2	11	15	1	3	5	14
9	13	14	16	3	4	12	14	6	11	12	15	5	12	13	14

SYSTEM NUMBER 38

1	2	3	16	4	8	10	12	2	5	6	14	2	4	7	12
1	4	5	16	1	8	12	14	3	8	11	12	1	2	7	14
1	6	7	16	10	11	12	14	1	3	8	10	2	7	10	11
1	8	9	16	4	5	12	14	3	5	8	14	3	4	9	12
1	10	11	16	1	5	10	12	9	11	12	13	1	3	9	14
1	12	13	16	1	4	11	12	1	9	10	13	3	9	10	11
1	14	15	16	1	4	10	14	5	9	13	14	4	6	12	13
2	4	6	16	1	5	11	14	4	7	8	15	1	6	13	14
2	5	7	16	4	5	10	11	7	11	12	15	6	10	11	13
2	8	10	16	3	6	8	15	1	7	10	15	8	10	11	15
2	9	11	16	3	9	13	15	5	7	14	15	2	5	9	12
2	12	14	16	6	7	13	15	1	2	8	11	2	9	10	14
2	13	15	16	2	6	9	15	2	4	5	8	1	2	4	9
3	4	8	16	2	3	7	15	3	12	13	14	3	5	12	15
3	5	9	16	2	7	9	13	1	3	11	13	3	10	14	15
3	6	12	16	2	3	6	13	3	4	5	13	1	3	4	15
3	7	14	16	2	6	7	8	6	12	14	15	5	6	8	12
3	10	13	16	2	3	8	9	1	6	11	15	6	8	10	14
3	11	15	16	3	7	8	13	4	5	6	15	1	4	6	8
4	7	10	16	6	8	9	13	7	8	9	10	5	7	12	13
4	9	13	16	3	6	7	9	7	9	12	14	7	10	13	14
4	11	14	16	2	3	10	12	1	7	9	11	1	4	7	13
4	12	15	16	2	3	4	14	4	5	7	9	3	4	6	10
5	6	13	16	2	3	5	11	1	3	7	12	3	6	11	14
5	8	15	16	6	7	10	12	3	5	7	10	1	3	5	6
5	10	14	16	4	6	7	14	3	4	7	11	7	8	11	14
5	11	12	16	5	6	7	11	1	6	9	12	1	5	7	8
6	8	11	16	4	8	9	14	5	6	9	10	8	9	12	15
6	9	14	16	5	8	9	11	4	6	9	11	4	9	10	15
6	10	15	16	1	8	13	15	5	8	10	13	9	11	14	15
7	8	12	16	10	12	13	15	4	8	11	13	1	5	9	15
7	9	15	16	4	13	14	15	2	8	14	15	2	8	12	13
7	11	13	16	5	11	13	15	1	2	12	15	2	4	10	13
8	13	14	16	2	6	11	12	2	5	10	15	2	11	13	14
9	10	12	16	1	2	6	10	2	4	11	15	1	2	5	13

SYSTEM NUMBER 43

1	2	3	16	3	8	11	15	1	4	9	12	2	8	12	15
1	4	5	16	3	9	10	14	2	3	4	12	4	5	8	11
1	6	7	16	2	3	10	11	5	6	8	10	3	4	5	10
1	8	9	16	2	3	8	9	1	5	6	9	2	4	5	9
1	10	11	16	1	3	9	11	2	3	5	6	6	7	8	11
1	12	13	16	1	3	8	10	7	8	10	15	3	6	7	10
1	14	15	16	8	9	10	11	1	7	9	15	2	6	7	9
2	4	6	16	1	2	9	10	2	3	7	15	8	11	12	13
2	5	7	16	1	2	8	11	11	13	14	15	3	10	12	13
2	8	10	16	4	7	12	15	8	10	13	14	2	9	12	13
2	9	11	16	5	7	13	15	1	9	13	14	2	9	14	15
2	12	14	16	4	5	6	15	2	3	13	14	3	4	6	11
2	13	15	16	6	7	14	15	1	2	4	15	4	6	9	10
3	4	8	16	6	12	13	15	4	10	11	15	1	4	6	8
3	5	9	16	5	6	13	14	3	6	8	14	3	5	7	11
3	6	12	16	5	6	7	12	1	2	6	14	5	7	9	10
3	7	14	16	4	6	12	14	6	10	11	14	1	5	7	8
3	10	15	16	4	6	7	13	3	7	8	13	3	11	12	14
3	11	13	16	7	12	13	14	1	2	7	13	1	8	12	14
4	7	10	16	4	5	12	13	7	10	11	13	9	10	13	15
4	9	15	16	4	5	7	14	5	9	12	14	1	8	13	15
4	11	12	16	1	3	5	13	3	5	8	12	6	8	9	12
4	13	14	16	2	5	10	13	1	2	5	12	1	6	10	12
5	6	11	16	5	9	11	13	5	10	11	12	2	6	11	12
5	8	13	16	1	3	6	15	3	4	7	9	7	8	9	14
5	10	14	16	2	6	10	15	1	4	7	11	1	7	10	14
5	12	15	16	6	9	11	15	2	4	7	8	2	7	11	14
6	8	15	16	1	3	7	12	1	5	11	14	3	4	13	15
6	9	14	16	2	7	10	12	2	5	8	14	4	8	9	13
6	10	13	16	7	9	11	12	3	6	9	13	1	4	10	13
7	8	12	16	4	8	14	15	1	6	11	13	2	4	11	13
7	9	13	16	1	3	4	14	2	6	8	13	3	5	14	15
7	11	15	16	2	4	10	14	10	12	14	15	5	8	9	15
8	11	14	16	4	9	11	14	3	9	12	15	1	5	10	15
9	10	12	16	4	8	10	12	1	11	12	15	2	5	11	15

SYSTEM NUMBER 44

1	2	3	16	3	6	8	11	3	5	10	12	6	9	11	15
1	4	5	16	3	4	9	10	1	2	5	12	4	5	8	11
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6	8	13	16	1	10	12	15	1	5	8	13	3	4	6	13
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7	10	12	16	2	4	7	9	4	6	10	15	1	5	9	15
8	12	15	16	1	2	4	15	2	3	6	15	5	8	10	15
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SYSTEM NUMBER 45

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2	8	10	16	2	6	14	15	2	4	7	11	1	6	8	13
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2	12	14	16	3	6	9	15	1	2	8	11	1	2	5	14
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8	12	15	16	1	3	9	12	3	5	8	10	4	5	8	11
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SYSTEM NUMBER 46

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2	5	7	16	1	6	9	13	2	8	12	15	5	7	9	11
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2	9	11	16	2	3	11	12	1	2	7	12	1	8	10	14
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3	4	8	16	4	10	11	12	1	2	5	9	2	7	8	11
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3	7	15	16	4	5	10	14	1	4	9	10	3	5	7	8
3	10	13	16	2	5	10	12	8	11	12	13	3	5	6	13
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8	11	15	16	6	8	12	14	7	11	14	15	3	8	9	11
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SYSTEM NUMBER 48

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SYSTEM NUMBER 49

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SYSTEM NUMBER 50

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3	4	8	16	3	8	14	15	1	5	6	12	1	5	7	10
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SYSTEM NUMBER 51

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7	8	13	16	4	8	9	15	2	5	8	13	1	4	8	13
7	9	12	16	8	9	10	13	9	12	14	15	1	6	8	15
7	11	14	16	6	8	9	11	4	9	11	12	7	9	10	14
8	11	12	16	3	4	5	10	6	9	10	12	4	7	9	13
9	10	15	16	2	3	5	15	2	9	12	13	6	7	9	15

SYSTEM NUMBER 52

1	2	3	16	2	5	10	11	2	3	14	15	1	2	7	11
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1	6	7	16	10	11	13	15	6	5	11	12	1	7	10	13
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1	10	11	16	2	6	11	13	6	7	9	11	3	4	12	15
1	12	13	16	2	4	11	15	2	7	9	15	3	10	12	13
1	14	15	16	2	4	10	13	1	5	8	10	4	5	9	15
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2	5	7	16	2	6	10	15	1	6	8	11	2	8	11	14
2	8	10	16	1	3	5	14	1	2	8	15	4	8	14	15
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3	10	14	16	1	3	7	8	4	7	9	12	5	6	8	13
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4	9	14	16	1	7	9	14	6	7	12	15	6	9	12	13
4	10	15	16	1	3	11	13	2	7	12	13	3	5	13	15
4	11	12	16	1	3	10	15	1	11	12	15	2	3	4	5
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6	10	13	16	10	12	14	15	5	11	14	15	1	9	13	15
6	11	15	16	4	6	12	14	2	5	6	14	1	2	4	9
7	8	12	16	2	5	8	9	4	5	10	14	1	6	9	10
7	9	10	16	8	9	10	15	7	8	9	13	8	9	11	12
7	11	14	16	4	6	8	9	7	8	11	15	8	12	13	15
8	11	13	16	3	4	13	14	2	6	7	8	2	4	8	12
9	12	15	16	3	6	11	14	4	7	8	10	6	8	10	12

SYSTEM NUMBER 55

1	2	3	16	1	5	7	8	1	2	5	12	1	2	9	13
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1	12	13	16	1	8	10	12	1	4	9	12	4	7	12	14
1	14	15	16	1	7	10	13	5	6	7	14	1	4	13	14
2	4	6	16	10	11	12	13	6	10	13	14	6	8	10	15
2	5	7	16	1	7	11	12	6	8	11	14	6	7	12	15
2	8	10	16	3	5	14	15	1	6	12	14	1	6	13	15
2	9	11	16	2	3	6	14	1	3	7	14	1	2	8	14
2	12	14	16	2	9	14	15	3	8	12	14	2	7	13	14
2	13	15	16	3	4	9	14	3	11	13	14	2	10	11	14
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3	5	11	16	4	6	14	15	5	6	11	13	3	6	7	13
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3	10	14	16	2	5	6	15	2	4	9	10	4	10	11	15
4	7	9	16	3	6	9	15	1	2	4	7	5	7	9	13
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4	12	15	16	2	3	7	10	2	7	8	15	1	3	4	10
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5	9	12	16	4	5	7	10	3	7	8	9	1	5	10	15
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6	11	15	16	11	12	14	15	5	10	12	14	1	9	10	14
7	8	14	16	1	5	6	9	4	6	9	13	9	12	13	14
7	10	12	16	6	7	9	10	4	6	7	8	2	6	8	9
7	11	13	16	6	9	11	12	1	4	6	11	2	6	7	11
8	11	12	16	2	5	10	13	4	6	10	12	1	2	6	10
9	10	15	16	2	5	8	11	2	7	9	12	2	6	12	13

SYSTEM NUMBER 56

1	2	3	16	1	3	4	6	4	5	9	11	2	4	8	15
1	4	5	16	3	4	11	14	1	5	9	14	1	2	8	11
1	6	7	16	4	6	14	15	5	9	10	15	2	6	8	14
1	8	9	16	4	6	10	11	4	8	11	13	3	4	13	15
1	10	11	16	1	4	11	15	1	8	13	14	1	3	11	13
1	12	13	16	1	4	10	14	8	10	13	15	3	6	13	14
1	14	15	16	1	6	11	14	2	3	6	7	4	5	12	15
2	4	6	16	10	11	14	15	2	4	7	11	1	5	11	12
2	5	7	16	1	6	10	15	1	2	7	14	5	6	12	14
2	8	10	16	2	5	9	13	2	7	10	15	4	7	9	15
2	9	11	16	2	7	12	13	1	2	4	9	1	7	9	11
2	12	14	16	5	7	8	13	2	6	9	10	6	7	9	14
2	13	15	16	8	9	12	13	2	9	14	15	1	2	6	13
3	4	8	16	3	5	12	13	3	5	6	10	2	4	13	14
3	5	11	16	2	3	8	13	3	5	14	15	2	10	11	13
3	6	12	16	3	7	9	13	1	4	7	13	3	7	10	11
3	7	15	16	3	5	8	9	6	7	10	13	1	5	6	8
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3	10	13	16	2	7	8	9	3	8	11	12	5	8	10	11
4	7	14	16	5	7	9	12	1	4	8	12	1	6	9	12
4	9	13	16	2	5	8	12	6	8	10	12	4	9	12	14
4	10	15	16	2	3	11	15	8	12	14	15	9	10	11	12
4	11	12	16	2	3	10	14	2	4	10	12	1	3	8	10
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5	8	15	16	8	9	11	15	1	2	12	15	9	11	13	14
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6	8	13	16	11	12	13	15	1	3	9	15	2	3	4	5
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6	11	15	16	1	3	5	7	5	6	11	13	1	2	5	10
7	8	12	16	4	5	6	7	1	5	13	15	2	5	6	15
7	9	10	16	5	7	11	15	3	7	8	14	3	4	7	12
7	11	13	16	5	7	10	14	4	7	8	10	7	11	12	14
8	11	14	16	1	3	12	14	6	7	8	11	1	7	10	12
9	12	15	16	3	10	12	15	1	7	8	15	6	7	12	15

SYSTEM NUMBER 57

1	2	3	16	1	3	4	6	1	3	12	15	1	2	4	9
1	4	5	16	2	4	10	15	4	5	9	10	2	6	9	14
1	6	7	16	4	6	14	15	5	9	11	14	3	5	10	15
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1	12	13	16	1	4	10	14	8	11	13	14	7	10	13	15
1	14	15	16	1	6	10	15	1	8	13	15	6	7	13	14
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2	5	7	16	1	6	11	14	2	7	11	14	8	10	12	15
2	8	10	16	2	7	8	13	1	2	7	15	6	8	12	14
2	9	11	16	5	7	8	9	2	4	8	11	2	6	10	12
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2	13	15	16	3	7	8	12	2	6	8	15	3	4	9	15
3	4	8	16	3	7	9	13	3	4	11	13	3	6	9	10
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3	7	15	16	2	3	12	13	4	5	11	12	5	6	10	13
3	9	14	16	2	3	8	9	1	5	12	14	1	5	11	13
3	10	13	16	2	5	9	13	5	6	12	15	4	7	8	15
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4	9	13	16	2	5	8	12	4	7	9	11	1	7	8	11
4	11	14	16	2	3	10	14	1	7	9	14	1	3	8	10
4	12	15	16	2	3	11	15	6	7	9	15	3	8	14	15
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5	8	15	16	8	9	10	14	2	4	13	14	1	9	10	13
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6	11	15	16	1	3	5	7	4	5	8	14	2	5	14	15
7	8	14	16	4	5	6	7	5	8	10	11	2	5	6	11
7	9	12	16	5	7	10	14	2	9	12	15	2	4	7	12
7	11	13	16	5	7	11	15	1	6	9	12	1	7	10	12
8	11	12	16	3	4	10	12	4	9	12	14	7	12	14	15
9	10	15	16	3	11	12	14	9	10	11	12	6	7	11	12

SYSTEM NUMBER 60

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1	6	7	16	6	7	13	15	2	4	8	14	2	3	6	14
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1	12	13	16	2	6	8	15	2	8	9	12	4	7	8	12
1	14	15	16	2	7	8	13	9	10	12	13	4	12	13	15
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2	5	7	16	2	7	10	15	1	5	6	15	5	7	8	9
2	8	10	16	3	9	11	14	1	2	5	8	5	9	13	15
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3	5	15	16	3	5	9	12	2	3	4	7	3	5	7	13
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3	7	11	16	1	3	5	14	3	4	10	15	4	6	8	9
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3	10	14	16	1	3	11	12	8	11	12	14	2	4	9	10
4	7	14	16	1	4	11	14	2	7	11	14	7	11	12	13
4	9	15	16	3	4	12	14	6	11	13	14	2	10	11	12
4	11	13	16	1	3	8	13	10	11	14	15	3	7	10	12
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7	8	15	16	4	5	8	13	5	12	13	14	1	4	8	15
7	9	12	16	4	5	6	10	5	6	7	14	5	6	11	12
7	10	13	16	4	5	7	15	2	5	14	15	5	7	10	11
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SYSTEM NUMBER 70

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8	13	14	16
11	12	15	16

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SYSTEM NUMBER 74

1	2	3	16	2	3	4	9	2	4	8	11	1	6	9	14
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1	14	15	16	2	3	5	14	7	10	14	15	2	5	8	12
2	4	6	16	5	7	11	14	1	3	4	13	3	8	11	12
2	5	7	16	2	3	7	11	1	5	9	13	9	10	13	14
2	8	10	16	1	6	10	12	1	2	11	13	2	5	10	13
2	9	12	16	6	12	13	15	1	7	13	14	3	10	11	13
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2	13	15	16	4	6	8	12	1	4	11	14	1	5	10	14
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3	5	11	16	6	8	10	15	6	7	9	13	4	5	13	14
3	6	12	16	1	6	8	13	6	11	13	14	2	4	7	13
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3	9	13	16	1	4	8	10	7	8	9	15	5	6	8	14
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4	7	15	16	1	10	13	15	5	10	12	15	3	9	12	15
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6	8	11	16	9	11	13	15	5	6	10	11	4	9	12	13
6	9	15	16	5	7	13	15	3	6	7	10	3	5	12	13
6	10	14	16	3	13	14	15	2	8	9	13	2	12	13	14
7	8	12	16	1	2	4	12	5	8	11	13	7	11	12	13
7	9	11	16	1	9	11	12	3	7	8	13	4	9	10	15
7	10	13	16	1	5	7	12	4	12	14	15	3	4	5	10
8	13	14	16	1	3	12	14	4	5	11	12	2	4	10	14
11	12	15	16	4	5	8	9	3	4	7	12	4	7	10	11

SYSTEM NUMBER 75

1	2	3	16	1	3	4	6	3	8	10	13	1	2	8	11
1	4	5	16	2	6	11	12	6	7	11	15	2	4	8	13
1	6	7	16	4	6	12	13	1	7	12	15	3	6	12	15
1	8	9	16	4	6	10	11	7	10	13	15	1	3	11	15
1	10	11	16	1	6	11	13	6	9	11	14	3	4	13	15
1	12	13	16	1	6	10	12	1	9	12	14	5	6	9	12
1	14	15	16	1	4	11	12	9	10	13	14	1	5	9	11
2	4	6	16	10	11	12	13	2	3	4	5	4	5	9	13
2	5	7	16	1	4	10	13	1	2	5	12	6	7	12	14
2	8	10	16	2	7	8	9	2	5	10	13	1	7	11	14
2	9	12	16	2	7	14	15	2	6	13	14	4	7	13	14
2	11	14	16	5	7	8	14	1	2	10	14	1	2	4	15
2	13	15	16	5	7	9	15	2	4	12	14	2	6	10	15
3	4	8	16	3	7	8	15	3	6	9	13	3	5	11	12
3	5	13	16	3	7	9	14	1	3	9	10	3	5	6	10
3	6	14	16	3	5	14	15	3	4	9	12	1	4	7	9
3	7	12	16	3	5	8	9	6	7	8	13	7	9	11	12
3	9	11	16	2	3	9	15	1	7	8	10	6	7	9	10
3	10	15	16	2	3	8	14	4	7	8	12	1	4	8	14
4	7	15	16	2	5	9	14	2	5	11	15	8	11	12	14
4	9	14	16	8	9	14	15	5	6	13	15	6	8	10	14
4	10	12	16	2	3	10	11	1	5	10	15	1	3	13	14
4	11	13	16	2	3	12	13	4	5	12	15	3	4	11	14
5	6	11	16	4	6	8	9	1	2	6	9	3	10	12	14
5	8	15	16	8	9	10	11	2	4	9	10	1	9	13	15
5	9	10	16	8	9	12	13	2	9	11	13	4	9	11	15
5	12	14	16	4	6	14	15	3	4	7	10	9	10	12	15
6	8	12	16	10	11	14	15	3	7	11	13	2	3	6	7
6	9	15	16	12	13	14	15	1	5	6	14	1	2	7	13
6	10	13	16	1	3	5	7	4	5	10	14	2	4	7	11
7	8	11	16	4	5	6	7	5	11	13	14	2	7	10	12
7	9	13	16	5	7	10	11	2	8	12	15	2	5	6	8
7	10	14	16	5	7	12	13	1	6	8	15	1	5	8	13
8	13	14	16	3	6	8	11	4	8	10	15	4	5	8	11
11	12	15	16	1	3	8	12	8	11	13	15	5	8	10	12

SYSTEM NUMBER 76

1	2	3	16	1	2	10	13	7	9	12	13	2	4	8	15
1	4	5	16	4	7	10	13	1	4	9	12	2	3	13	14
1	6	7	16	2	6	7	13	5	6	9	12	3	4	7	14
1	8	9	16	2	4	5	13	7	11	13	14	1	3	6	14
1	10	11	16	1	5	7	13	1	4	11	14	2	8	12	13
1	12	13	16	1	4	6	13	5	6	11	14	4	7	8	12
1	14	15	16	1	2	4	7	2	3	10	15	1	6	8	12
2	4	6	16	4	5	6	7	3	7	13	15	2	9	11	13
2	5	7	16	1	2	5	6	1	3	4	15	4	7	9	11
2	8	10	16	3	8	9	15	3	5	6	15	1	6	9	11
2	9	12	16	9	11	12	15	3	5	8	13	1	6	10	15
2	11	14	16	8	12	14	15	1	3	7	8	1	3	11	13
2	13	15	16	3	11	14	15	2	3	6	8	3	4	5	11
3	4	8	16	9	10	14	15	5	9	10	13	2	3	7	11
3	5	14	16	10	11	12	14	1	7	9	10	1	8	13	14
3	6	11	16	3	8	10	14	2	6	9	10	4	5	8	14
3	7	9	16	3	9	10	11	5	12	13	14	2	7	8	14
3	10	13	16	8	9	10	12	1	7	12	14	1	9	13	15
3	12	15	16	8	9	11	14	2	6	12	14	4	5	9	15
4	7	15	16	3	8	11	12	4	10	11	15	2	7	9	15
4	9	10	16	3	9	12	14	5	11	13	15	4	5	10	12
4	11	13	16	4	8	9	13	1	7	11	15	2	7	10	12
4	12	14	16	2	5	8	9	2	6	11	15	3	4	6	10
5	6	13	16	6	7	8	9	3	6	9	13	3	5	7	10
5	8	12	16	2	5	10	11	1	3	5	9	1	2	9	14
5	9	11	16	6	7	10	11	2	3	4	9	4	6	9	14
5	10	15	16	4	13	14	15	6	10	13	14	5	7	9	14
6	8	14	16	2	5	14	15	1	5	10	14	10	12	13	15
6	9	15	16	6	7	14	15	2	4	10	14	1	2	12	15
6	10	12	16	1	3	10	12	6	11	12	13	4	6	12	15
7	8	13	16	3	4	12	13	1	5	11	12	5	7	12	15
7	10	14	16	2	3	5	12	2	4	11	12	8	10	11	13
7	11	12	16	3	6	7	12	7	8	10	15	1	2	8	11
8	11	15	16	1	4	8	10	6	8	13	15	4	6	8	11
9	13	14	16	5	6	8	10	1	5	8	15	5	7	8	11