School of Mathematics and Statistics
Faculty of Science, Technology, Engineering and Mathematics

2018 PhD Projects

Project title	Infinite antichains in small permutation classes
Principal supervisor	Robert Brignall
Second supervisor	Jozef Širáň
Discipline	Pure mathematics
Research area/keywords	permutation patterns, well-quasi-ordering, growth rates
Suitable for	Full time applicants

Project background and description

For a general background to "pattern avoiding" permutation classes, see Vatter's recent survey [4].

In the study of permutation classes, a famous result due to Marcus and Tardos asserts that $\limsup_{n\to\infty} \sqrt[n]{c_n}$ is bounded by a constant, where c_n denotes the number of permutations of length n in a given class. The smallest upper bound is the *upper growth rate*, and it remains an open problem in general whether $\sqrt[n]{c_n}$ has a true limit (the *growth rate* of the class). In two important papers, Vatter [2] and [3] classify all possible growth rates of permutation classes below $\xi \approx 2.30522$. Central to the study, and causing considerable trouble, is the appearance of infinite antichains inside some permutation classes at and above growth rate $\kappa \approx 2.20557$.

This PhD proposal aims to study the *well-quasi-ordered* permutation classes (i.e. those that do not contain infinite antichains) in the range below growth rate 4. Two key questions arise:

- What infinite antichains do we encounter below growth rate 4, and at what growth rates?
- Below growth rate 4, what are the possible growth rates of well-quasi-ordered permutation classes?

Typical elements of the three conjectured 'smallest' antichains (i.e. those occurring in the classes with smallest growth rates) are shown in Figure 1. The leftmost of these antichains is known to be the smallest, first appearing in classes with growth rate κ.

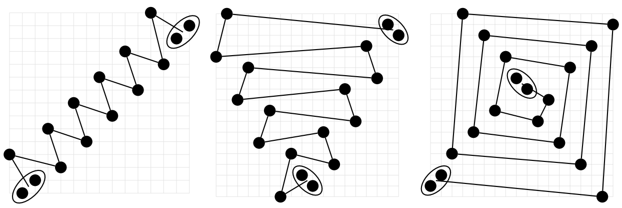

Figure 1: Typical elements from the three 'smallest' infinite permutation antichains.

Background reading/references

[1] Brignall, R., Grid classes and partial well order. *J. Comb. Theor. Ser. A*, 119 (2012), 99–116. Preprint version: https://arxiv.org/abs/0906.3723

[2] Vatter, V., Growth rates of permutation classes: from countable to uncountable, *J. Lond. Math. Soc.*, accepted. Preprint version: https://arxiv.org/abs/1605.04297

[3] Vatter, V., Small permutation classes, *Proc. Lond. Math. Soc.* (3), 103 (2011), 879–921. Preprint version: https://arxiv.org/abs/0712.4006

[4] Vatter, V. Permutation classes. In *Handbook of enumerative combinatorics*, Discrete Math. Appl. 538 (Boca Raton). CRC Press, Boca Raton, FL, 2015, pp. 753–833. Preprint version: https://arxiv.org/abs/1409.5159.