Simple Permutations

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Introduction

1. Basic Concepts
   - Permutation Classes
   - Intervals and Simple Permutations

2. Algebraic Generating Functions for Sets of Permutations
   - Finitely Many Simples
   - Sets of Permutations

3. A Decomposition Theorem with Enumerative Consequences
   - Aim
   - Pin Sequences
   - Decomposing Simple Permutations

4. Decidability and Unavoidable Structures
   - More on Pins
   - Decidability
Outline

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   - More on Pins
   - Decidability
Basic Concepts
Permutation Classes

Pattern Involvement

- Regard a permutation of length \( n \) as an ordering of the symbols 1, \ldots, \( n \).

- A permutation \( \tau = t_1 t_2 \ldots t_k \) is involved in the permutation \( \sigma = s_1 s_2 \ldots s_n \) if there exists a subsequence \( s_{i_1}, s_{i_2}, \ldots, s_{i_k} \) order isomorphic to \( \tau \).

Example
Basic Concepts
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Example

\[
\begin{array}{c}
1 & 3 & 5 & 2 & 4 \\
4 & 2 & 1 & 6 & 3 & 8 & 5 & 7
\end{array}
\]
Basic Concepts
Permutation Classes

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Example

1 3 5 2 4 $\prec$ 4 2 1 6 3 8 5 7
Permutation Classes

- Involvement forms a partial order on the set of all permutations.
- Downsets of permutations in this partial order form permutation classes.
- A permutation class $C$ can be seen to avoid certain permutations. Write $C = \text{Av}(B)$.

Example

The class $C = \text{Av}(12)$ consists of all the decreasing permutations:

$$\{1, 21, 321, 4321, \ldots\}$$
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Generating Functions

- $C_n$ – permutations in $C$ of length $n$.
- $\sum |C_n| x^n$ is the generating function.

Example

The generating function of $C = Av(12)$ is:

$$1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$$
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Pick any permutation $\pi$.

An interval of $\pi$ is a set of contiguous indices $I = [a, b]$ such that $\pi(I) = \{\pi(i) : i \in I\}$ is also contiguous.

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Only intervals are *singletons* and the *whole thing*. 

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Basic Concepts
Intervals and Simple Permutations

Simple Permutations

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**Example**

![Graph showing intervals and permutations](image-url)
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Example
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Special Simple Permutations

- Parallel alternations.
- Wedge permutations
- Two flavours of wedge simple permutation.
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Special Simple Permutations

- Parallel alternations.
- **Wedge** permutations – not simple!
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Motivation

Example

- 132-avoiders: generic structure.
- Only simple permutations are 1, 12, and 21.
- Enumerate recursively: $f(x) = xf(x)^2 + 1$. 
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— Bousquet-Mélou, 2006

- We can always write permutations with a simple block pattern, the substitution decomposition.
- Use recursive enumeration for classes with finitely many simple permutations.
- Expect an algebraic generating function.
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Expect an algebraic generating function.
Theorem (RB, SH, VV)

In a permutation class $\mathcal{C}$ with only finitely many simple permutations, the following sequences have algebraic generating functions:

- the number of permutations in $\mathcal{C}_n$ (Albert and Atkinson),
- the number of alternating permutations in $\mathcal{C}_n$,
- the number of even permutations in $\mathcal{C}_n$,
- the number of Dumont permutations in $\mathcal{C}_n$,
- the number of permutations in $\mathcal{C}_n$ avoiding any finite set of blocked or barred permutations,
- the number of involutions in $\mathcal{C}_n$, and
- Any (finite) combination of the above.
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A Decomposition Theorem with Enumerative Consequences

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Aim

What we want to find

- Large simple permutation, size $f(k)$.
- Find two simple permutations inside, each of size $k$.
- Overlap of at most two points – almost disjoint.
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Why I

- Every simple of length $\geq 4$ contains 132.
- Every simple of length $\geq f(4)$ contains 2 almost disjoint copies of 132.
- $\geq f(f(4))$ contains 4 copies of 132.

Theorem (Bóna; Mansour and Vainshtein)

For every fixed $r$, the class of all permutations containing at most $r$ copies of 132 has an algebraic generating function.
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Why II

Av(β_1^{≤r_1}, β_2^{≤r_2}, \ldots, β_k^{≤r_k}) — the class with: \leq r_1 copies of β_1, \leq r_2 copies of β_2, etc.

Corollary

If the class Av(β_1, β_2, \ldots, β_k) contains only finitely many simple permutations then for all choices of nonnegative integers r_1, r_2, \ldots, and r_k, the class Av(β_1^{≤r_1}, β_2^{≤r_2}, \ldots, β_k^{≤r_k}) also contains only finitely many simple permutations.

Corollary

For all r and s, every subclass of Av(2413^{≤r}, 3142^{≤s}) contains only finitely many simple permutations and thus has an algebraic generating function.
A Decomposition Theorem with Enumerative Consequences

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\[ \text{Av}(\beta_1^{\leq r_1}, \beta_2^{\leq r_2}, \ldots, \beta_k^{\leq r_k}) \] — the class with: \( \leq r_1 \) copies of \( \beta_1 \), \( \leq r_2 \) copies of \( \beta_2 \), etc.

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Start with any two points.

Extend up, down, left, or right – this is a right pin.

A proper pin must be maximal and cut the previous pin, but not the rectangle.

A right-reaching pin sequence.
Proper Pin Sequences

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A right-reaching pin sequence.
The points of the proper pin sequence form a simple permutation.
A Decomposition Theorem with Enumerative Consequences
Decomposing Simple Permutations

A Technical Theorem

**Theorem**

*Every simple permutation of length at least* $2(2048k^8)^{(2048k^8)^{(2k)}}$
*contains either a proper pin sequence of length at least* $2k$
*or a parallel alternation or a wedge simple permutation of length at least* $2k$.

- Proper pin sequence $\Rightarrow$ two almost disjoint simples.
- Parallel alternation $\Rightarrow$ two almost disjoint simples.
- Wedge simple permutation $\Rightarrow$ two almost disjoint simples.
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Decomposing Simple Permutations

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- **Wedge simple** permutation $\Rightarrow$ two almost disjoint simples.
The Decomposition Theorem

Theorem (RB, SH, VV)

There is a function $f(k)$ such that every simple permutation of length at least $f(k)$ contains two simple subsequences, each of length at least $k$, which share at most two entries in common.
Decidability and Unavoidable Structures

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The Language of Pins

- Encode as: 1
- Pattern involvement $\leftrightarrow$ partial order on pin words.
- Avoiding a pattern $\leftrightarrow$ avoiding every pin word generating that pattern.
The Language of Pins

- Encode as: 11
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The Language of Pins

Encode as: 11R

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- Encode as: 11RU
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The Language of Pins

- Encode as: 11RUL
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The Language of Pins

- Encode as: 11RULD
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The Language of Pins

- Encode as: 11RULDR
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Decidability

Theorem (RB, NR, VV)

*It is decidable whether a finitely based permutation class contains only finitely many simple permutations.*

Proof.

- Technical theorem $\Rightarrow$ only look for arbitrary parallel or wedge simple permutations, or proper pin sequences.
- Parallel and wedge simple permutations easily verified.
- Proper pin sequences $\leftrightarrow$ the language of pins.
- Language of pins avoiding a given pattern is regular.
- Decidable if a regular language is infinite.
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Proof.

- Technical theorem $\implies$ only look for arbitrary parallel or wedge simple permutations, or proper pin sequences.
  - Parallel and wedge simple permutations easily verified.
  - Proper pin sequences $\leftrightarrow$ the language of pins.
  - Language of pins avoiding a given pattern is regular.
  - Decidable if a regular language is infinite.
Áttu eitthvað ódýrara?
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*Do you have anything cheaper?*