Simple Permutations

R.L.F. Brignall
joint work with Sophie Huczynska, Nik Ruškuc and Vincent Vatter

School of Mathematics and Statistics
University of St Andrews

Thursday 15th June, 2006

Introduction

Outline

Pattern Involvement

- Regard a permutation of length n as an ordering of the symbols $1, \ldots, n$.
- A permutation $\tau = t_1 t_2 \ldots t_k$ is involved in the permutation $\sigma = s_1 s_2 \ldots s_n$ if there exists a subsequence $s_{i_1}, s_{i_2}, \ldots, s_{i_k}$ order isomorphic to τ.

Example

Pattern Involvement

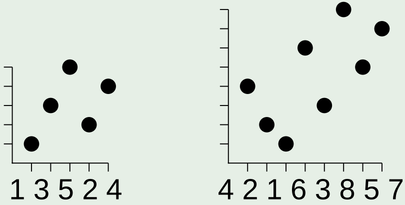
- Regard a permutation of length n as an ordering of the symbols $1, \ldots, n$.
- A permutation $\tau = t_1 t_2 \ldots t_k$ is involved in the permutation $\sigma = s_1 s_2 \ldots s_n$ if there exists a subsequence $s_{i_1}, s_{i_2}, \ldots, s_{i_k}$ order isomorphic to τ.

Example

Pattern Involvement

- Regard a permutation of length n as an ordering of the symbols $1, \ldots, n$.
- A permutation $\tau = t_1 t_2 \ldots t_k$ is involved in the permutation $\sigma = s_1 s_2 \ldots s_n$ if there exists a subsequence $s_{i_1}, s_{i_2}, \ldots, s_{i_k}$ order isomorphic to τ.

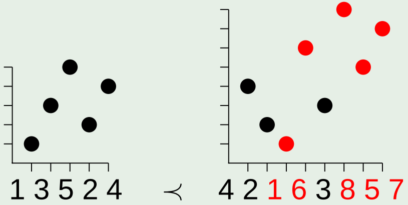
Example



Pattern Involvement

- Regard a permutation of length n as an ordering of the symbols $1, \ldots, n$.
- A permutation $\tau = t_1 t_2 \ldots t_k$ is involved in the permutation $\sigma = s_1 s_2 \ldots s_n$ if there exists a subsequence $s_{i_1}, s_{i_2}, \ldots, s_{i_k}$ order isomorphic to τ.

Example



Permutation Classes

- Involvement forms a partial order on the set of all permutations.
- Downsets of permutations in this partial order form permutation classes.
- A permutation class $\mathcal{C}$ can be seen to avoid certain permutations. Write $\mathcal{C} = \mathrm{Av}(B)$.

Example

The class $\mathcal{C} = \mathrm{Av}(12)$ consists of all the decreasing permutations:

$$\{1, 21, 321, 4321, \ldots\}$$

Permutation Classes

- Involvement forms a partial order on the set of all permutations.
- Downsets of permutations in this partial order form permutation classes.
- A permutation class $\mathcal{C}$ can be seen to avoid certain permutations. Write $\mathcal{C} = \mathrm{Av}(B)$.

Example

The class $\mathcal{C} = \mathrm{Av}(12)$ consists of all the decreasing permutations:

$$\{1, 21, 321, 4321, \ldots\}$$

Permutation Classes

- Involvement forms a partial order on the set of all permutations.
- Downsets of permutations in this partial order form permutation classes.
- A permutation class $\mathcal{C}$ can be seen to avoid certain permutations. Write $\mathcal{C} = \mathrm{Av}(B)$.

Example

The class $\mathcal{C} = \mathrm{Av}(12)$ consists of all the decreasing permutations:

$$\{1, 21, 321, 4321, \ldots\}$$

Permutation Classes

- Involvement forms a partial order on the set of all permutations.
- Downsets of permutations in this partial order form permutation classes.
- A permutation class $\mathcal{C}$ can be seen to avoid certain permutations. Write $\mathcal{C} = \mathrm{Av}(B)$.

Example

The class $\mathcal{C} = \mathrm{Av}(12)$ consists of all the decreasing permutations:

$$\{1, 21, 321, 4321, \ldots\}$$

Permutation Classes

- Involvement forms a partial order on the set of all permutations.
- Downsets of permutations in this partial order form permutation classes.
- A permutation class $\mathcal{C}$ can be seen to avoid certain permutations. Write $\mathcal{C} = \mathrm{Av}(B)$.

Example

The class $\mathcal{C} = \mathrm{Av}(12)$ consists of all the decreasing permutations:

$$\{1, 21, 321, 4321, \ldots\}$$

Generating Functions

- $\mathcal{C}_n$ – permutations in $\mathcal{C}$ of length n.
- $\sum |\mathcal{C}_n| x^n$ is the generating function.

Example

The generating function of $\mathcal{C} = \mathrm{Av}(12)$ is:

$$1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$$

Generating Functions

- $\mathcal{C}_n$ – permutations in $\mathcal{C}$ of length n.
- $\sum |\mathcal{C}_n| x^n$ is the generating function.

Example

The generating function of $\mathcal{C} = \mathrm{Av}(12)$ is:

$$1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$$

Generating Functions

- $\mathcal{C}_n$ – permutations in $\mathcal{C}$ of length n.
- $\sum |\mathcal{C}_n| x^n$ is the generating function.

Example

The generating function of $\mathcal{C} = \mathrm{Av}(12)$ is:

$$1 + x + x^2 + x^3 + \cdots \quad = \quad \frac{1}{1-x}$$

Generating Functions

- $\mathcal{C}_n$ – permutations in $\mathcal{C}$ of length n.
- $\sum |\mathcal{C}_n| x^n$ is the generating function.

Example

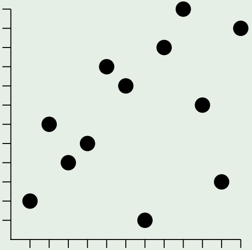
The generating function of $\mathcal{C} = \mathrm{Av}(12)$ is:

$$1 + x + x^2 + x^3 + \cdots \quad = \quad \frac{1}{1-x}$$

Intervals

- Pick any permutation π.
- An interval of π is a set of contiguous indices $I = [a, b]$ such that $\pi(I) = \{\pi(i) : i \in I\}$ is also contiguous.

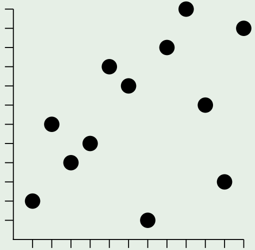
Example



Intervals

- Pick any permutation π.
- An interval of π is a set of contiguous indices $I = [a, b]$ such that $\pi(I) = \{\pi(i) : i \in I\}$ is also contiguous.

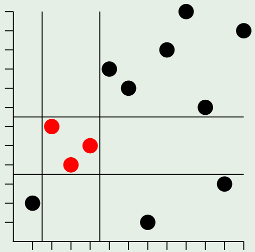
Example



Intervals

- Pick any permutation π.
- An interval of π is a set of contiguous indices $I = [a, b]$ such that $\pi(I) = \{\pi(i) : i \in I\}$ is also contiguous.

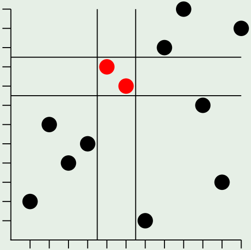
Example



Intervals

- Pick any permutation π.
- An interval of π is a set of contiguous indices $I = [a, b]$ such that $\pi(I) = \{\pi(i) : i \in I\}$ is also contiguous.

Example



Simple Permutations

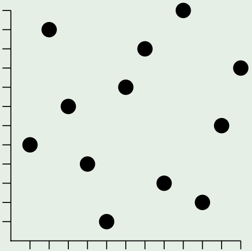
- Only intervals are singletons and the whole thing.

Example

Simple Permutations

- Only intervals are singletons and the whole thing.

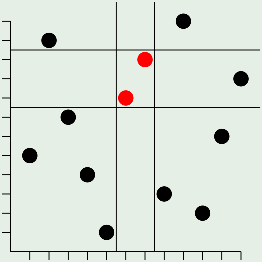
Example



Simple Permutations

- Only intervals are singletons and the whole thing.

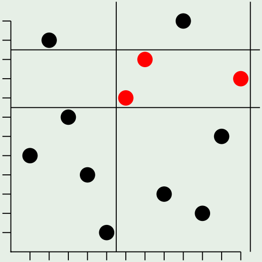
Example



Simple Permutations

- Only intervals are singletons and the whole thing.

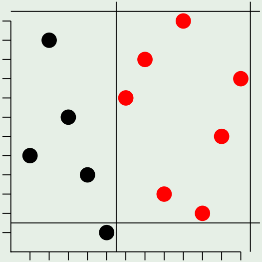
Example



Simple Permutations

- Only intervals are singletons and the whole thing.

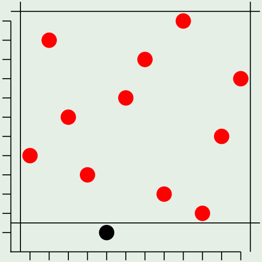
Example



Simple Permutations

- Only intervals are singletons and the whole thing.

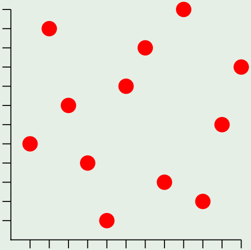
Example



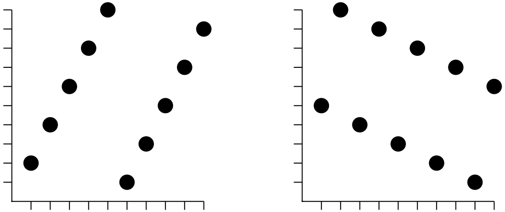
Simple Permutations

- Only intervals are singletons and the whole thing.

Example

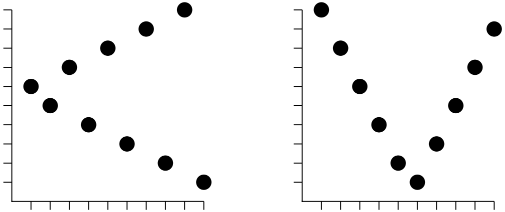


Special Simple Permutations



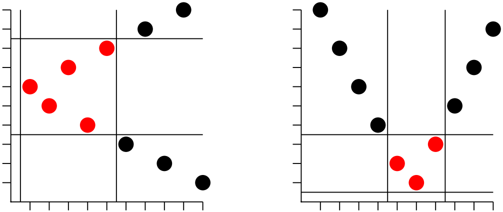
- Parallel alternations.
- Wedge permutations
- Two flavours of wedge simple permutation.

Special Simple Permutations



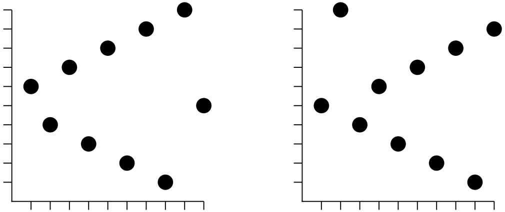
- Parallel alternations.
- Wedge permutations
- Two flavours of wedge simple permutation.

Special Simple Permutations



- Parallel alternations.
- Wedge permutations – not simple!
- Two flavours of wedge simple permutation.

Special Simple Permutations

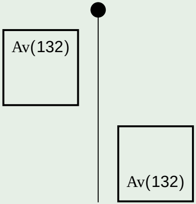


- Parallel alternations.
- Wedge permutations
- Two flavours of wedge simple permutation.

Outline

Motivation

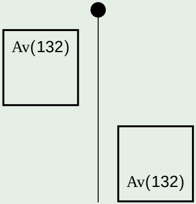
Example



- 132-avoiders: generic structure.
- Only simple permutations are1, 12, and 21.
- Enumerate recursively: $f(x) = xf(x)^2 + 1$.

Motivation

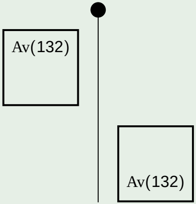
Example



- 132-avoiders: generic structure.
- Only simple permutations are1, 12, and 21.
- Enumerate recursively: $f(x) = xf(x)^2 + 1$.

Motivation

Example



- 132-avoiders: generic structure.
- Only simple permutations are 1, 12, and 21.
- Enumerate recursively: $f(x) = xf(x)^2 + 1$.

Motivation II

"...the standard intuition of what a family with an algebraic generating function looks like: the algebraicity suggests that it may (or should...), be possible to give a recursive description of the objects based on disjoint union of sets and concatentation of objects."

— Bousquet-Mélou, 2006

- We can always write permutations with a simple block pattern, the substitution decomposition.
- Use recursive enumeration for classes with finitely many simple permutations.
- Expect an algebraic generating function.

Motivation II

"...the standard intuition of what a family with an algebraic generating function looks like: the algebraicity suggests that it may (or should...), be possible to give a recursive description of the objects based on disjoint union of sets and concatentation of objects."

— Bousquet-Mélou, 2006

- We can always write permutations with a simple block pattern, the substitution decomposition.
- Use recursive enumeration for classes with finitely many simple permutations.
- Expect an algebraic generating function.

Motivation II

> *"...the standard intuition of what a family with an algebraic generating function looks like: the algebraicity suggests that it may (or should...), be possible to give a recursive description of the objects based on disjoint union of sets and concatentation of objects."*

— Bousquet-Mélou, 2006

- We can always write permutations with a simple block pattern, the substitution decomposition.
- Use recursive enumeration for classes with finitely many simple permutations.
- Expect an algebraic generating function.

Motivation II

"...the standard intuition of what a family with an algebraic generating function looks like: the algebraicity suggests that it may (or should...), be possible to give a recursive description of the objects based on disjoint union of sets and concatentation of objects."

— Bousquet-Mélou, 2006

- We can always write permutations with a simple block pattern, the substitution decomposition.
- Use recursive enumeration for classes with finitely many simple permutations.
- Expect an algebraic generating function.

Algebraic Generating Functions

Theorem (RB, SH, VV)

In a permutation class $\mathcal{C}$ with only finitely many simple permutations, the following sequences have algebraic generating functions:

- *the number of permutations in $\mathcal{C}_n$ (Albert and Atkinson),*
- *the number of alternating permutations in $\mathcal{C}_n$,*
- *the number of even permutations in $\mathcal{C}_n$,*
- *the number of Dumont permutations in $\mathcal{C}_n$,*
- *the number of permutations in $\mathcal{C}_n$ avoiding any finite set of blocked or barred permutations,*
- *the number of involutions in $\mathcal{C}_n$, and*
- *Any (finite) combination of the above.*

Algebraic Generating Functions

Theorem (RB, SH, VV)

In a permutation class $\mathcal{C}$ with only finitely many simple permutations, the following sequences have algebraic generating functions:

- *the number of permutations in $\mathcal{C}_n$ (Albert and Atkinson),*
- *the number of alternating permutations in $\mathcal{C}_n$,*
- *the number of even permutations in $\mathcal{C}_n$,*
- *the number of Dumont permutations in $\mathcal{C}_n$,*
- *the number of permutations in $\mathcal{C}_n$ avoiding any finite set of blocked or barred permutations,*
- *the number of involutions in $\mathcal{C}_n$, and*
- *Any (finite) combination of the above.*

Algebraic Generating Functions

Theorem (RB, SH, VV)

In a permutation class $\mathcal{C}$ with only finitely many simple permutations, the following sequences have algebraic generating functions:

- *the number of permutations in $\mathcal{C}_n$ (Albert and Atkinson),*
- *the number of alternating permutations in $\mathcal{C}_n$,*
- *the number of even permutations in $\mathcal{C}_n$,*
- *the number of Dumont permutations in $\mathcal{C}_n$,*
- *the number of permutations in $\mathcal{C}_n$ avoiding any finite set of blocked or barred permutations,*
- *the number of involutions in $\mathcal{C}_n$, and*
- *Any (finite) combination of the above.*

Algebraic Generating Functions

Theorem (RB, SH, VV)

In a permutation class $\mathcal{C}$ with only finitely many simple permutations, the following sequences have algebraic generating functions:

- *the number of permutations in $\mathcal{C}_n$ (Albert and Atkinson),*
- *the number of alternating permutations in $\mathcal{C}_n$,*
- *the number of even permutations in $\mathcal{C}_n$,*
- *the number of Dumont permutations in $\mathcal{C}_n$,*
- *the number of permutations in $\mathcal{C}_n$ avoiding any finite set of blocked or barred permutations,*
- *the number of involutions in $\mathcal{C}_n$, and*
- *Any (finite) combination of the above.*

Algebraic Generating Functions

Theorem (RB, SH, VV)

In a permutation class $\mathcal{C}$ with only finitely many simple permutations, the following sequences have algebraic generating functions:

- *the number of permutations in $\mathcal{C}_n$ (Albert and Atkinson),*
- *the number of alternating permutations in $\mathcal{C}_n$,*
- *the number of even permutations in $\mathcal{C}_n$,*
- *the number of Dumont permutations in $\mathcal{C}_n$,*
- *the number of permutations in $\mathcal{C}_n$ avoiding any finite set of blocked or barred permutations,*
- *the number of involutions in $\mathcal{C}_n$, and*
- *Any (finite) combination of the above.*

Algebraic Generating Functions

Theorem (RB, SH, VV)

In a permutation class $\mathcal{C}$ with only finitely many simple permutations, the following sequences have algebraic generating functions:

- *the number of permutations in $\mathcal{C}_n$ (Albert and Atkinson),*
- *the number of alternating permutations in $\mathcal{C}_n$,*
- *the number of even permutations in $\mathcal{C}_n$,*
- *the number of Dumont permutations in $\mathcal{C}_n$,*
- *the number of permutations in $\mathcal{C}_n$ avoiding any finite set of blocked or barred permutations,*
- *the number of involutions in $\mathcal{C}_n$, and*
- *Any (finite) combination of the above.*

Algebraic Generating Functions

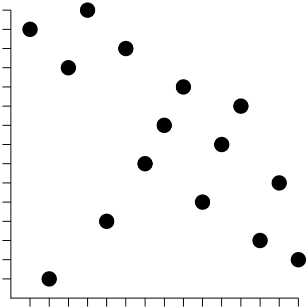
Theorem (RB, SH, VV)

In a permutation class $\mathcal{C}$ with only finitely many simple permutations, the following sequences have algebraic generating functions:

- *the number of permutations in $\mathcal{C}_n$ (Albert and Atkinson),*
- *the number of alternating permutations in $\mathcal{C}_n$,*
- *the number of even permutations in $\mathcal{C}_n$,*
- *the number of Dumont permutations in $\mathcal{C}_n$,*
- *the number of permutations in $\mathcal{C}_n$ avoiding any finite set of blocked or barred permutations,*
- *the number of involutions in $\mathcal{C}_n$, and*
- *Any (finite) combination of the above.*

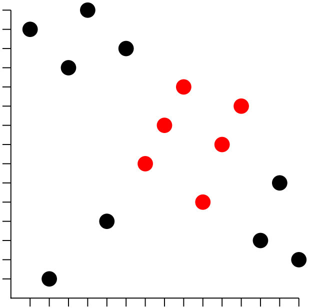
Outline

What we want to find



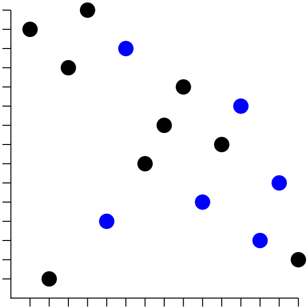
- Large simple permutation, size $f(k)$.
- Find two simple permutations inside, each of size k.
- Overlap of at most two points – almost disjoint.

What we want to find



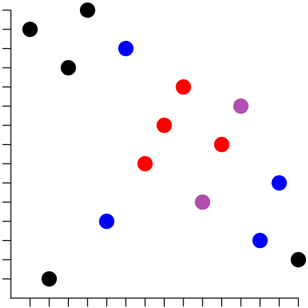
- Large simple permutation, size $f(k)$.
- Find two simple permutations inside, each of size k.
- Overlap of at most two points – almost disjoint.

What we want to find



- Large simple permutation, size $f(k)$.
- Find two simple permutations inside, each of size k.
- Overlap of at most two points – almost disjoint.

What we want to find



- Large simple permutation, size $f(k)$.
- Find two simple permutations inside, each of size k.
- Overlap of at most two points – almost disjoint.

Why I

- Every simple of length ≥ 4 contains 132.
- Every simple of length $\geq f(4)$ contains 2 almost disjoint copies of 132.
- $\geq f(f(4))$ contains 4 copies of 132.

$$\vdots$$

Theorem (Bóna; Mansour and Vainshtein)

For every fixed r, the class of all permutations containing at most r copies of 132 has an algebraic generating function.

Why I

- Every simple of length ≥ 4 contains 132.
- Every simple of length $\geq f(4)$ contains 2 almost disjoint copies of 132.
- $\geq f(f(4))$ contains 4 copies of 132.

⋮

Theorem (Bóna; Mansour and Vainshtein)

For every fixed r, the class of all permutations containing at most r copies of 132 has an algebraic generating function.

Why I

- Every simple of length ≥ 4 contains 132.
- Every simple of length $\geq f(4)$ contains 2 almost disjoint copies of 132.
- $\geq f(f(4))$ contains 4 copies of 132.

$$\vdots$$

Theorem (Bóna; Mansour and Vainshtein)

For every fixed r, the class of all permutations containing at most r copies of 132 has an algebraic generating function.

Why I

- Every simple of length ≥ 4 contains 132.
- Every simple of length $\geq f(4)$ contains 2 almost disjoint copies of 132.
- $\geq f(f(4))$ contains 4 copies of 132.

$$\vdots$$

Theorem (Bóna; Mansour and Vainshtein)

For every fixed r, the class of all permutations containing at most r copies of 132 has an algebraic generating function.

Why II

- $\mathrm{Av}(\beta_1^{\leq r_1}, \beta_2^{\leq r_2}, \ldots, \beta_k^{\leq r_k})$ — the class with: $\leq r_1$ copies of β_1, $\leq r_2$ copies of β_2, etc.

Corollary

If the class $\mathrm{Av}(\beta_1, \beta_2, \ldots, \beta_k)$ contains only finitely many simple permutations then for all choices of nonnegative integers r_1, r_2, $\ldots$, and r_k, the class $\mathrm{Av}(\beta_1^{\leq r_1}, \beta_2^{\leq r_2}, \ldots, \beta_k^{\leq r_k})$ also contains only finitely many simple permutations.

Corollary

For all r and s, every subclass of $\mathrm{Av}(2413^{\leq r}, 3142^{\leq s})$ contains only finitely many simple permutations and thus has an algebraic generating function.

Why II

- $\mathrm{Av}(\beta_1^{\leq r_1}, \beta_2^{\leq r_2}, \ldots, \beta_k^{\leq r_k})$ — the class with: $\leq r_1$ copies of β_1, $\leq r_2$ copies of β_2, etc.

Corollary

If the class $\mathrm{Av}(\beta_1, \beta_2, \ldots, \beta_k)$ contains only finitely many simple permutations then for all choices of nonnegative integers r_1, r_2, $\ldots$, and r_k, the class $\mathrm{Av}(\beta_1^{\leq r_1}, \beta_2^{\leq r_2}, \ldots, \beta_k^{\leq r_k})$ also contains only finitely many simple permutations.

Corollary

For all r and s, every subclass of $\mathrm{Av}(2413^{\leq r}, 3142^{\leq s})$ contains only finitely many simple permutations and thus has an algebraic generating function.

Why II

- $\mathrm{Av}(\beta_1^{\leq r_1}, \beta_2^{\leq r_2}, \ldots, \beta_k^{\leq r_k})$ — the class with: $\leq r_1$ copies of β_1, $\leq r_2$ copies of β_2, etc.

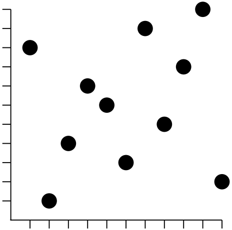
Corollary

If the class $\mathrm{Av}(\beta_1, \beta_2, \ldots, \beta_k)$ contains only finitely many simple permutations then for all choices of nonnegative integers r_1, r_2, $\ldots$, and r_k, the class $\mathrm{Av}(\beta_1^{\leq r_1}, \beta_2^{\leq r_2}, \ldots, \beta_k^{\leq r_k})$ also contains only finitely many simple permutations.

Corollary

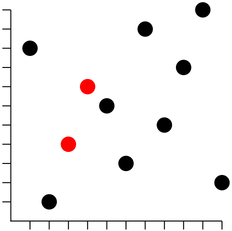
For all r and s, every subclass of $\mathrm{Av}(2413^{\leq r}, 3142^{\leq s})$ contains only finitely many simple permutations and thus has an algebraic generating function.

Proper Pin Sequences



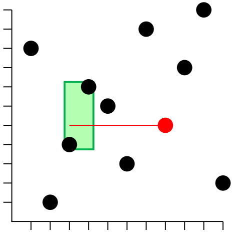
- Start with any two points.
- Extend up, down, left, or right – this is a right pin.
- A proper pin must be maximal and cut the previous pin, but not the rectangle.
- A right-reaching pin sequence.

Proper Pin Sequences



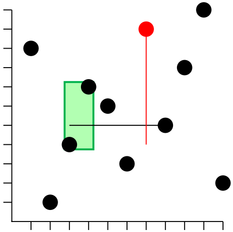
- Start with any two points.
- Extend up, down, left, or right – this is a right pin.
- A proper pin must be maximal and cut the previous pin, but not the rectangle.
- A right-reaching pin sequence.

Proper Pin Sequences



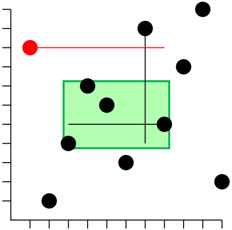
- Start with any two points.
- Extend up, down, left, or right – this is a right pin.
- A proper pin must be maximal and cut the previous pin, but not the rectangle.
- A right-reaching pin sequence.

Proper Pin Sequences



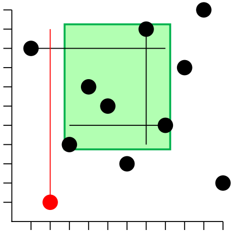
- Start with any two points.
- Extend up, down, left, or right – this is a right pin.
- A proper pin must be maximal and cut the previous pin, but not the rectangle.
- A right-reaching pin sequence.

Proper Pin Sequences



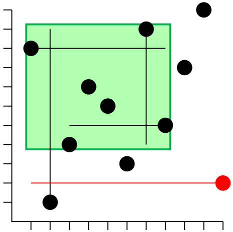
- Start with any two points.
- Extend up, down, left, or right – this is a right pin.
- A proper pin must be maximal and cut the previous pin, but not the rectangle.
- A right-reaching pin sequence.

Proper Pin Sequences



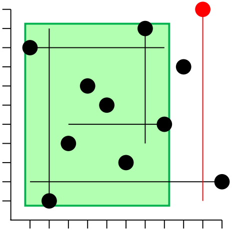
- Start with any two points.
- Extend up, down, left, or right – this is a right pin.
- A proper pin must be maximal and cut the previous pin, but not the rectangle.
- A right-reaching pin sequence.

Proper Pin Sequences



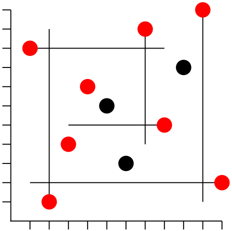
- Start with any two points.
- Extend up, down, left, or right – this is a right pin.
- A proper pin must be maximal and cut the previous pin, but not the rectangle.
- A right-reaching pin sequence.

Proper Pin Sequences



- Start with any two points.
- Extend up, down, left, or right – this is a right pin.
- A proper pin must be maximal and cut the previous pin, but not the rectangle.
- A right-reaching pin sequence.

Simples and Pin Sequences



- The points of the proper pin sequence form a simple permutation.

A Technical Theorem

Theorem

Every simple permutation of length at least $2(2048k^8)^{(2048k^8)^{(2k)}}$ contains either a proper pin sequence of length at least $2k$ or a parallel alternation or a wedge simple permutation of length at least $2k$.

- Proper pin sequence $\Rightarrow$ two almost disjoint simples.
- Parallel alternation $\Rightarrow$ two almost disjoint simples.
- Wedge simple permutation $\Rightarrow$ two almost disjoint simples.

A Technical Theorem

Theorem

Every simple permutation of length at least $2(2048k^8)^{(2048k^8)^{(2k)}}$ contains either a proper pin sequence of length at least $2k$ or a parallel alternation or a wedge simple permutation of length at least $2k$.

- Proper pin sequence $\Rightarrow$ two almost disjoint simples.
- Parallel alternation $\Rightarrow$ two almost disjoint simples.
- Wedge simple permutation $\Rightarrow$ two almost disjoint simples.

A Technical Theorem

Theorem

Every simple permutation of length at least $2(2048k^8)^{(2048k^8)^{(2k)}}$ contains either a proper pin sequence of length at least $2k$ or a parallel alternation or a wedge simple permutation of length at least $2k$.

- Proper pin sequence $\Rightarrow$ two almost disjoint simples.
- Parallel alternation $\Rightarrow$ two almost disjoint simples.
- Wedge simple permutation $\Rightarrow$ two almost disjoint simples.

A Technical Theorem

Theorem

Every simple permutation of length at least $2(2048k^8)^{(2048k^8)^{(2k)}}$ contains either a proper pin sequence of length at least $2k$ or a parallel alternation or a wedge simple permutation of length at least $2k$.

- Proper pin sequence $\Rightarrow$ two almost disjoint simples.
- Parallel alternation $\Rightarrow$ two almost disjoint simples.
- Wedge simple permutation $\Rightarrow$ two almost disjoint simples.

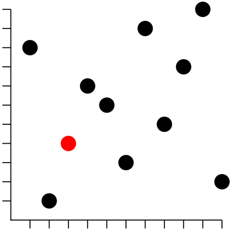
The Decomposition Theorem

Theorem (RB, SH, VV)

There is a function $f(k)$ such that every simple permutation of length at least $f(k)$ contains two simple subsequences, each of length at least k, which share at most two entries in common.

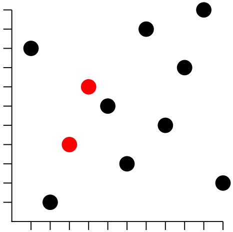
Outline

The Language of Pins



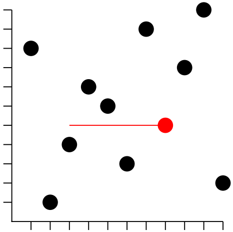
- Encode as: 1
- Pattern involvement ↔ partial order on pin words.
- Avoiding a pattern ↔ avoiding every pin word generating that pattern.

The Language of Pins



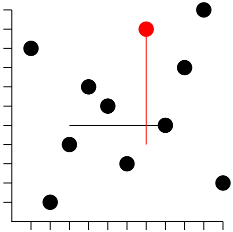
- Encode as: 11
- Pattern involvement ↔ partial order on pin words.
- Avoiding a pattern ↔ avoiding every pin word generating that pattern.

The Language of Pins



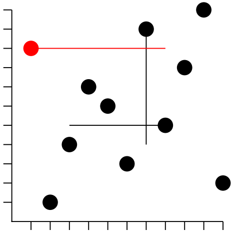
- Encode as: 11R
- Pattern involvement ↔ partial order on pin words.
- Avoiding a pattern ↔ avoiding every pin word generating that pattern.

The Language of Pins



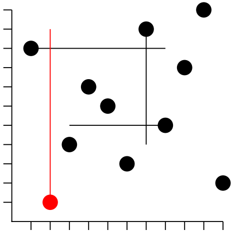
- Encode as: 11RU
- Pattern involvement $\leftrightarrow$ partial order on pin words.
- Avoiding a pattern $\leftrightarrow$ avoiding every pin word generating that pattern.

The Language of Pins



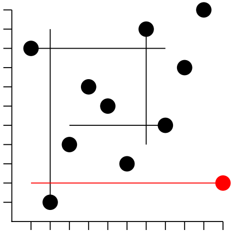
- Encode as: 11RUL
- Pattern involvement $\leftrightarrow$ partial order on pin words.
- Avoiding a pattern $\leftrightarrow$ avoiding every pin word generating that pattern.

The Language of Pins



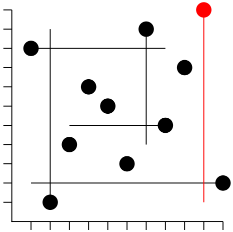
- Encode as: 11RULD
- Pattern involvement ↔ partial order on pin words.
- Avoiding a pattern ↔ avoiding every pin word generating that pattern.

The Language of Pins



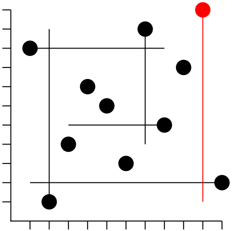
- Encode as: 11RULDR
- Pattern involvement $\leftrightarrow$ partial order on pin words.
- Avoiding a pattern $\leftrightarrow$ avoiding every pin word generating that pattern.

The Language of Pins



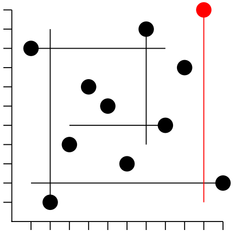
- Encode as: 11RULDRU
- Pattern involvement ↔ partial order on pin words.
- Avoiding a pattern ↔ avoiding every pin word generating that pattern.

The Language of Pins



- Encode as: 11RULDRU
- Pattern involvement $\leftrightarrow$ partial order on pin words.
- Avoiding a pattern $\leftrightarrow$ avoiding every pin word generating that pattern.

The Language of Pins



- Encode as: 11RULDRU
- Pattern involvement ↔ partial order on pin words.
- Avoiding a pattern ↔ avoiding every pin word generating that pattern.

Decidability

Theorem (RB, NR, VV)

It is decidable whether a finitely based permutation class contains only finitely many simple permutations.

Proof.

- Technical theorem ⇒ only look for arbitrary parallel or wedge simple permutations, or proper pin sequences.
- Parallel and wedge simple permutations easily verified.
- Proper pin sequences ⟷ the language of pins.
- Language of pins avoiding a given pattern is regular.
- Decidable if a regular language is infinite.

□

Decidability

Theorem (RB, NR, VV)

It is decidable whether a finitely based permutation class contains only finitely many simple permutations.

Proof.

- Technical theorem $\Rightarrow$ only look for arbitrary parallel or wedge simple permutations, or proper pin sequences.
- Parallel and wedge simple permutations easily verified.
- Proper pin sequences $\leftrightarrow$ the language of pins.
- Language of pins avoiding a given pattern is regular.
- Decidable if a regular language is infinite.

□

Decidability

Theorem (RB, NR, VV)

It is decidable whether a finitely based permutation class contains only finitely many simple permutations.

Proof.

- Technical theorem $\Rightarrow$ only look for arbitrary parallel or wedge simple permutations, or proper pin sequences.
- Parallel and wedge simple permutations easily verified.
- Proper pin sequences $\leftrightarrow$ the language of pins.
- Language of pins avoiding a given pattern is regular.
- Decidable if a regular language is infinite.

□

Decidability

Theorem (RB, NR, VV)

It is decidable whether a finitely based permutation class contains only finitely many simple permutations.

Proof.

- Technical theorem $\Rightarrow$ only look for arbitrary parallel or wedge simple permutations, or proper pin sequences.
- Parallel and wedge simple permutations easily verified.
- Proper pin sequences $\leftrightarrow$ the language of pins.
- Language of pins avoiding a given pattern is regular.
- Decidable if a regular language is infinite.

□

Decidability

Theorem (RB, NR, VV)

It is decidable whether a finitely based permutation class contains only finitely many simple permutations.

Proof.

- Technical theorem $\Rightarrow$ only look for arbitrary parallel or wedge simple permutations, or proper pin sequences.
- Parallel and wedge simple permutations easily verified.
- Proper pin sequences $\leftrightarrow$ the language of pins.
- Language of pins avoiding a given pattern is regular.
- Decidable if a regular language is infinite.

□

Decidability

Theorem (RB, NR, VV)

It is decidable whether a finitely based permutation class contains only finitely many simple permutations.

Proof.

- Technical theorem $\Rightarrow$ only look for arbitrary parallel or wedge simple permutations, or proper pin sequences.
- Parallel and wedge simple permutations easily verified.
- Proper pin sequences $\leftrightarrow$ the language of pins.
- Language of pins avoiding a given pattern is regular.
- Decidable if a regular language is infinite.

□

Áttu eitthvað ódýrara?

Áttu eitthvað ódýrara?
Do you have anything cheaper?