Characterising inflations of monotone grid classes of permutations

Robert ~~Brignall~~ Nicolasson

Joint work wið Michæl Albert and Aistis Atminas

Reykjavik, 29þ June 2017

Two concepts of structure

Enumeration Structure Characterisation

Finitely many simple permutations

Theorem (Albert & Atkinson, 2005): Any permutation class containing only finitely many simple permutations has an algebraic generating function.

(Geometric) griddability

Theorem (Albert, Atkinson, Bouvel, Ruškuc & Vatter, 2013): Any permutation class that is geometrically griddable has a rational generating function.

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Theorem (Albert & Atkinson, 2005): Any permutation class containing only finitely many simple permutations is finitely based and well-quasi-ordered.

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Two concepts of structure

Enumeration Structure Characterisation

Finitely many simple permutations

B., Huczynska & Vatter, 2008: Characterisation of simples, giving…

Theorem (B., Ruškuc & Vatter, 2008): It is decidable whether a permutation class contains only finitely many simple permutations.

Bassino, Bouvel, Pierrot & Rossin, 2015: Efficient algorithm.

(Geometric) griddability

Theorem (Huczynska & Vatter, 2006): A permutation class is ~~geometrically~~ griddable if and only if it avoids long sums of 21 and skew sums of 12.

N.B. Reinstating 'geometrically' into the above seems hard!

Geometrically griddable simples

Theorem (Albert, Ruškuc & Vatter, 2015)

Any permutation class containing only geometrically griddable simples has an algebraic generating function, is finitely based, and is well-quasi-ordered.

- Related to this: Every class with growth rate $< \kappa \approx 2.20557\ldots$ has a rational generating function.

Geometrically griddable simples

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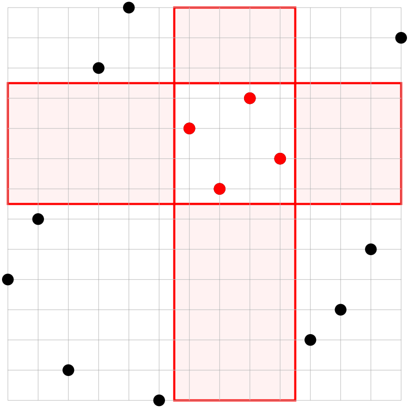
- Related to this: Every class with growth rate $< \kappa \approx 2.20557\ldots$ has a rational generating function.
- Today: when are the simple permutations in a class ~~geometrically~~ griddable?
- Equivalently: what are the 'minimal simple obstructions' to griddability?
- As before, reinstating 'geometrically' is out of range.

Philosophical aside

A permutation class $\mathcal{C}$ is deflatable if its simple permutations belong to a proper subclass $\mathcal{D} \subsetneq \mathcal{C}$.

Albert, Atkinson, Homberger, Pantone (2016).

You want a definition of simple?



None of these (except trivial).

And you want me to define ‘griddable’ *too*?!

- Mumble mumble …chopping permutations up …monotone cells …mumble mumble.

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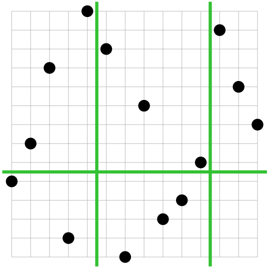

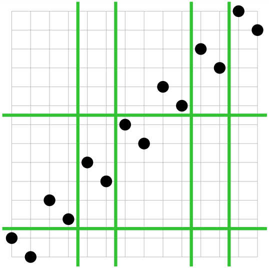

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- Mumble mumble ... chopping permutations up ... monotone cells ... mumble mumble.
- Actually, you don’t need to know. All you need is this:

Theorem (Huczynska & Vatter, 2006)

A class $\mathcal{C}$ is griddable if and only if it avoids long sums of 21 and skew sums of 12.

- Easy-to-check: $\mathrm{Av}(B)$ is griddable if and only if there exist $\beta, \gamma \in B$ such that:

$\beta \leq$  and $\gamma \leq$ 

Griddability of simples

The same thing holds (obviously) for simple permutations:

Proposition (Essentially Huczynska & Vatter)

The simple permutations in a class $\mathcal{C}$ are griddable if and only if they avoid long sums of 21 and skew sums of 12.

- Not easy-to-check: $\mathcal{C}$ can contain long sums of 21 without the simples doing so.

Griddability of simples

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Proposition (Essentially Huczynska & Vatter)

The simple permutations in a class $\mathcal{C}$ are griddable if and only if they avoid long sums of 21 and skew sums of 12.

Theorem (Albert, Atminas & B., 2017+)

The simple permutations in a class $\mathcal{C}$ are griddable if and only if $\mathcal{C}$ does not contain the following structures, or their symmetries:

- *arbitrarily long parallel sawtooth alternations,*
- *arbitrarily long sliced wedge sawtooth alternations,*
- *proper pin sequences with arbitrarily many turns, and*
- *spiral proper pin sequences with arbitrarily many extensions.*

The basic simples

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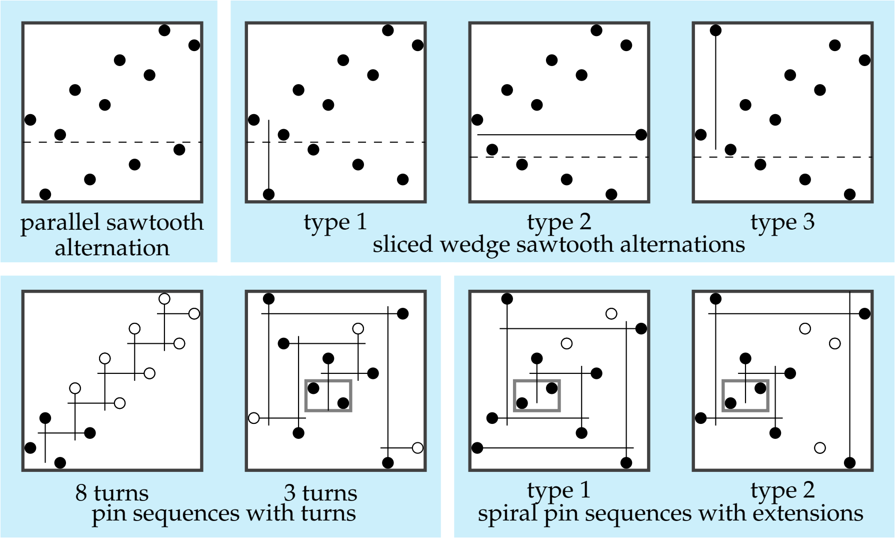

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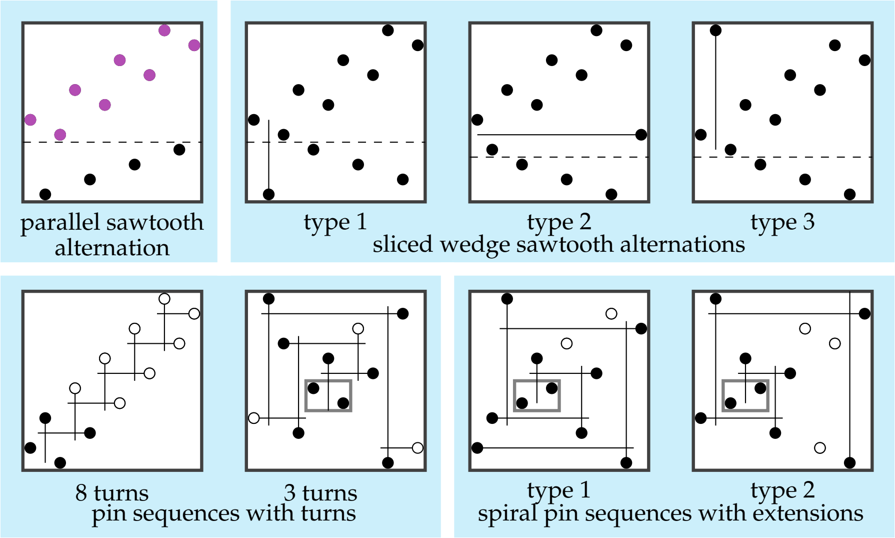

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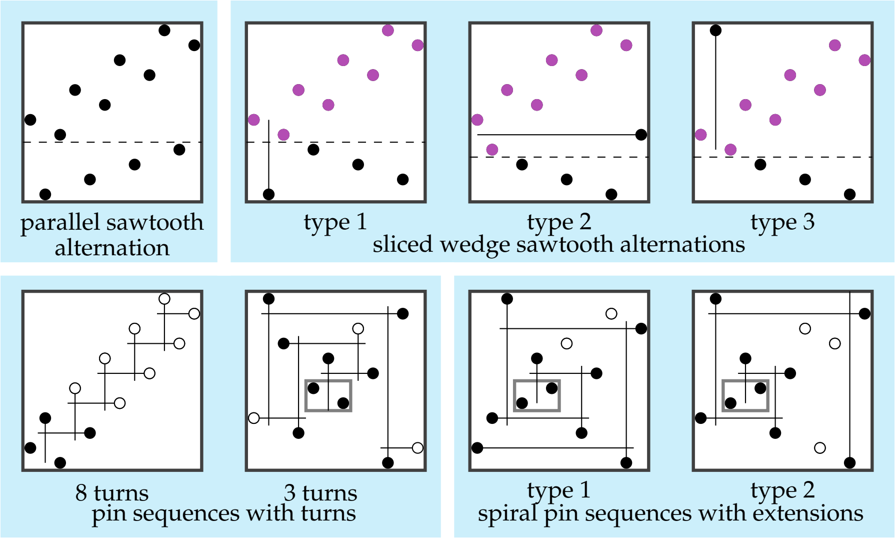

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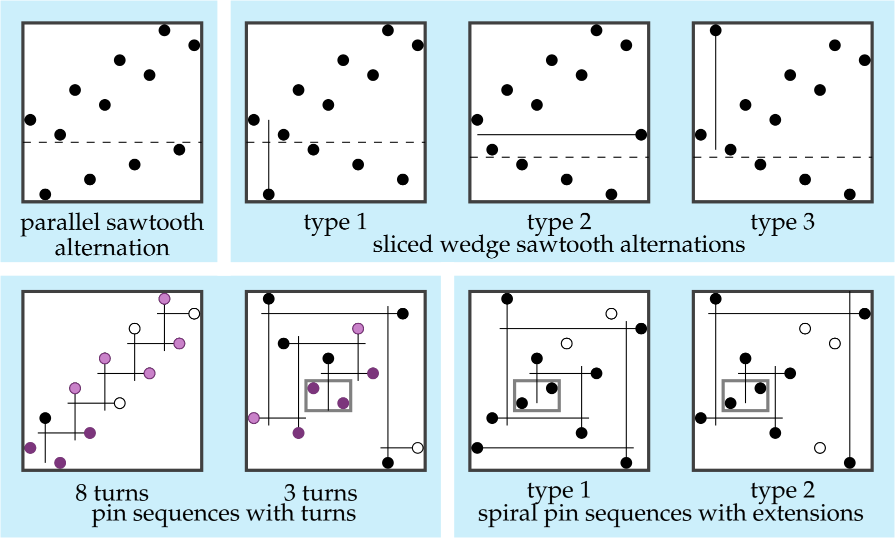

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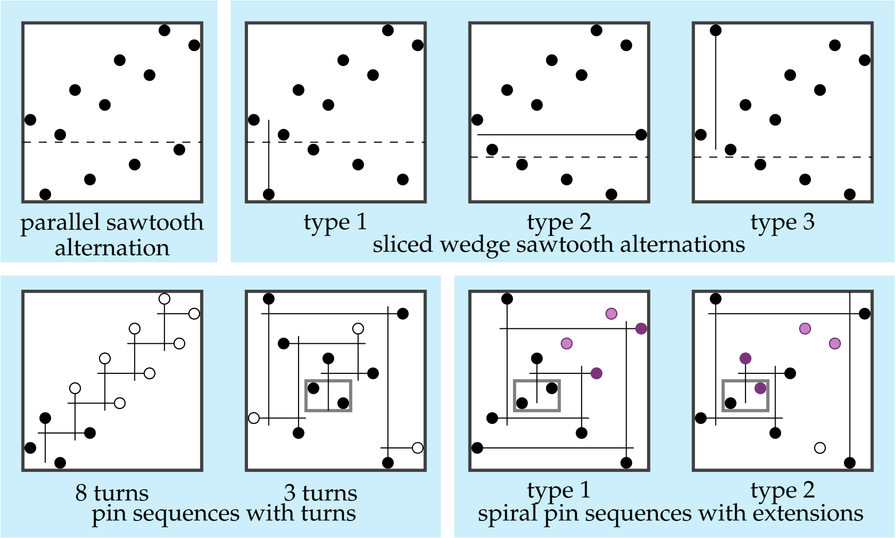
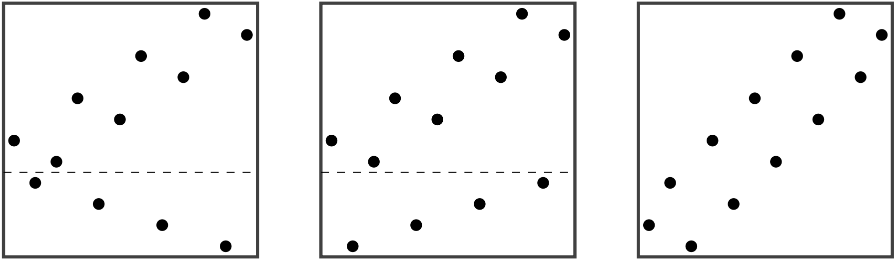


Step 1: An easier characterisation

Theorem

There exists a function $f(n)$ such that every simple permutation that contains a sum of $f(n)$ copies of 21 must contain a parallel or wedge sawtooth alternation of length $3n$ or an increasing oscillation of length n.



wedge sawtooth — parallel sawtooth — increasing oscillation

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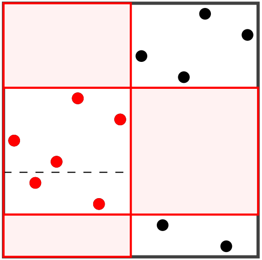
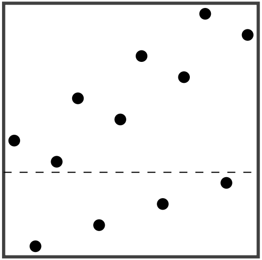
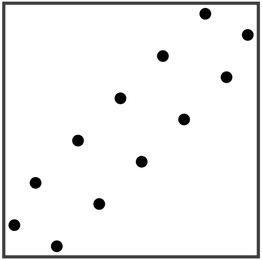


wedge sawtooth



parallel sawtooth



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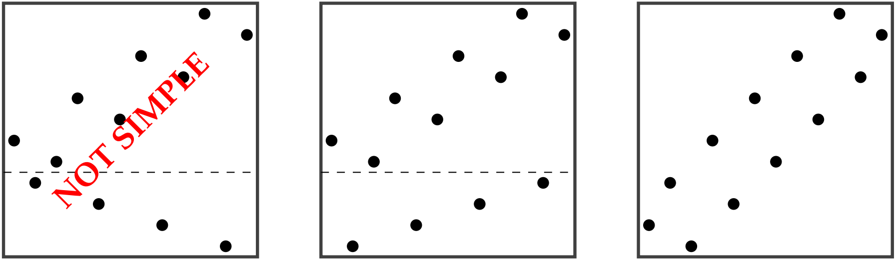
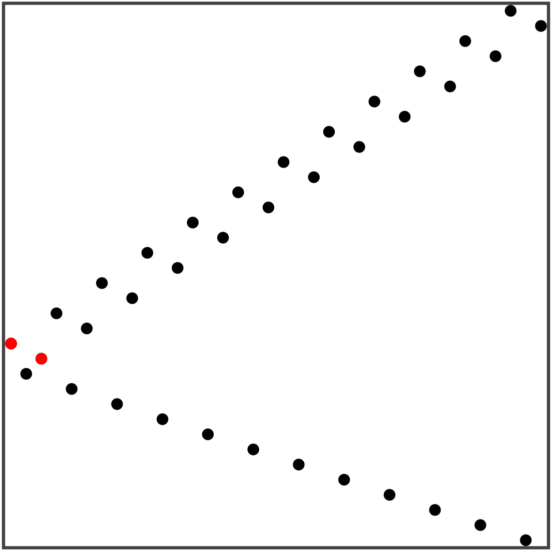


wedge sawtooth

parallel sawtooth

increasing oscillation

Step 2: Handle wedge sawtooths



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- Form a 'pin sequence'.
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- Otherwise: long pin sequence.

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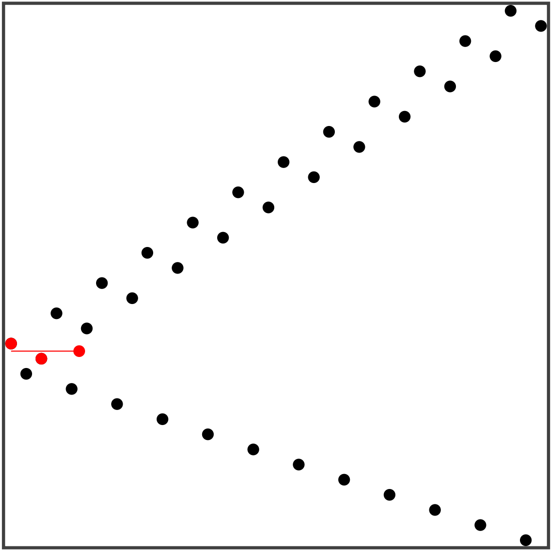

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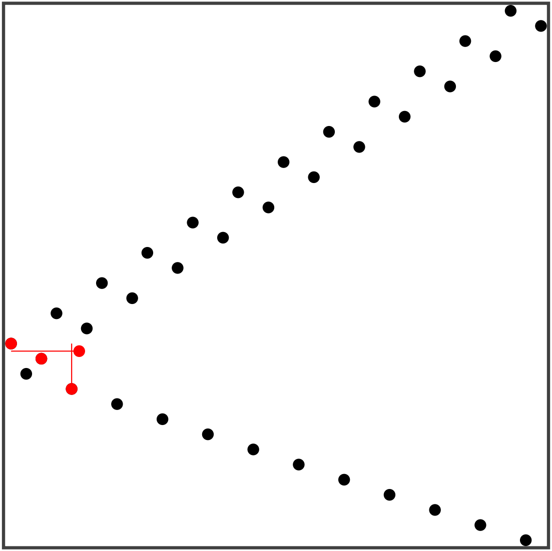

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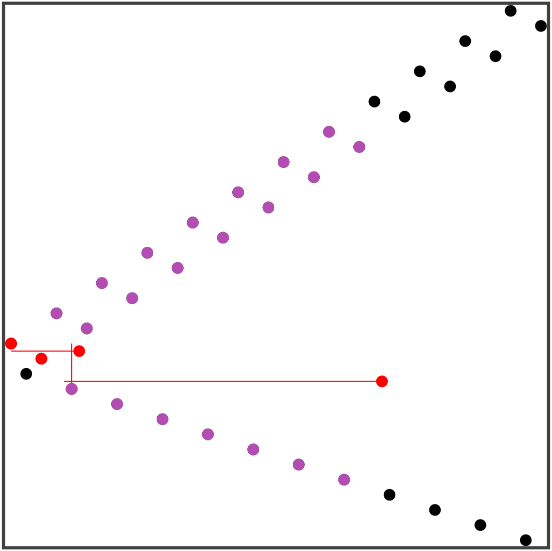

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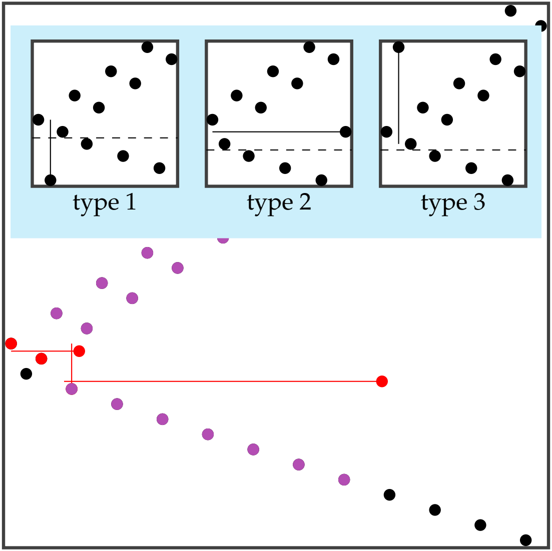

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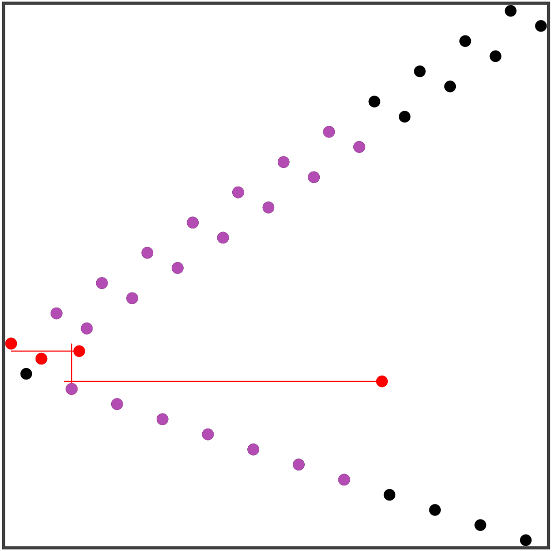

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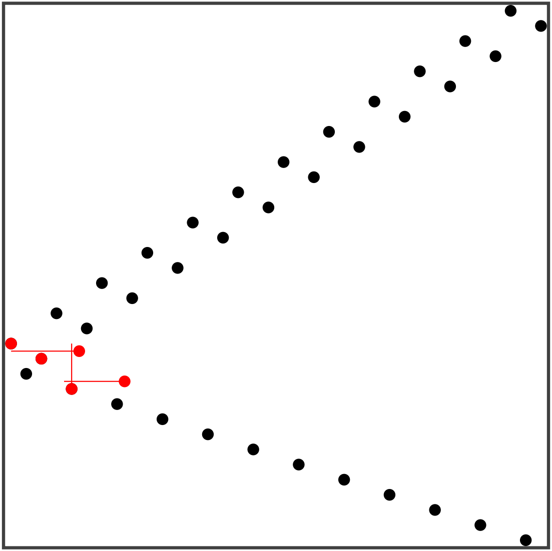

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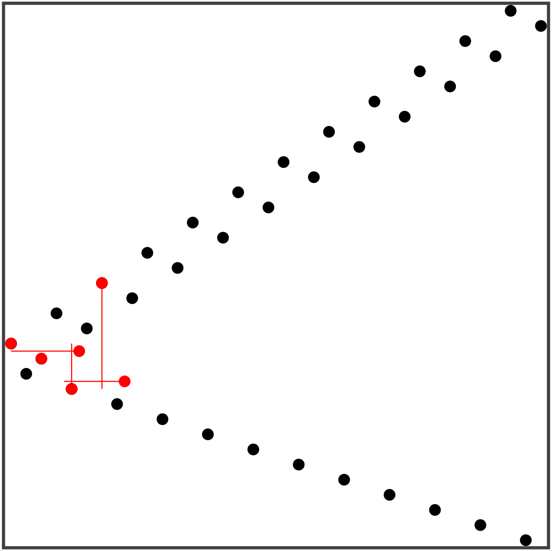

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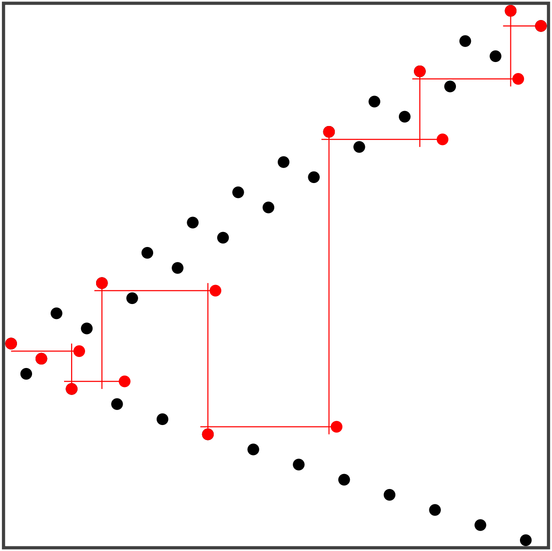

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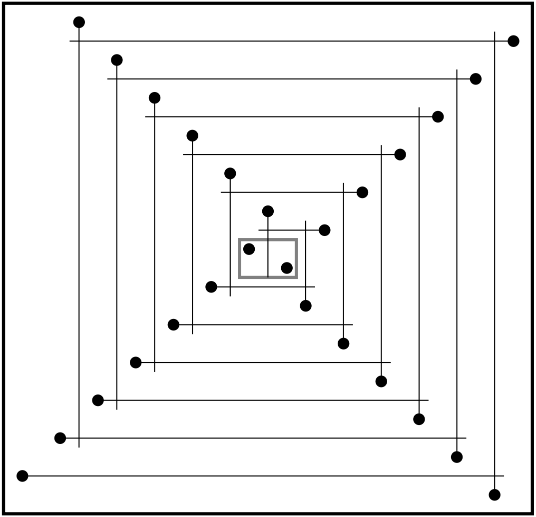

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- No turns = spiral pin sequences.
- Use wedge sawtooth to find ‘extensions’.

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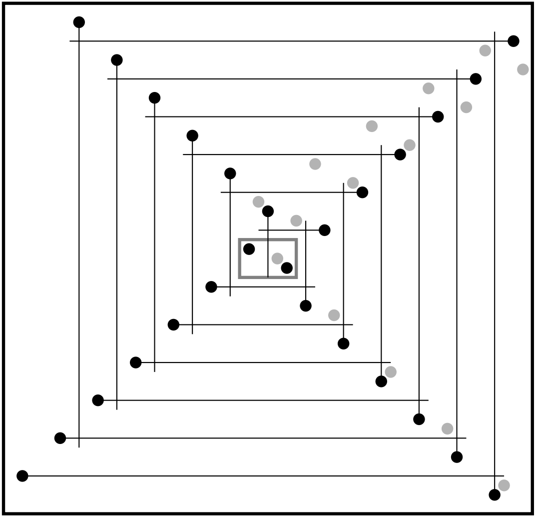

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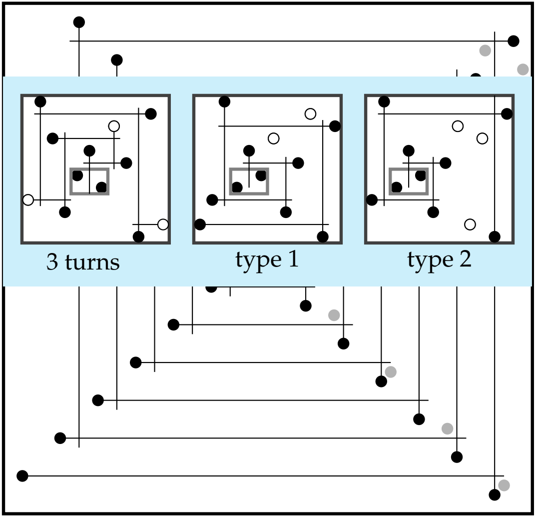

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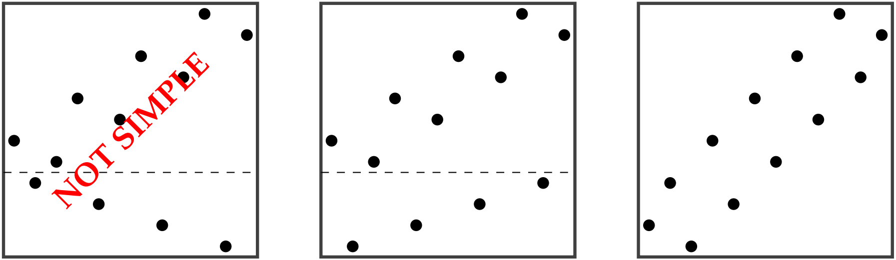
And so. . .

Theorem (I've already shown you this)

There exists a function $f(n)$ such that every simple permutation that contains a sum of $f(n)$ copies of 21 must contain a parallel or wedge sawtooth alternation of length $3n$ or an increasing oscillation of length n.



wedge sawtooth | parallel sawtooth | increasing oscillation

And so…

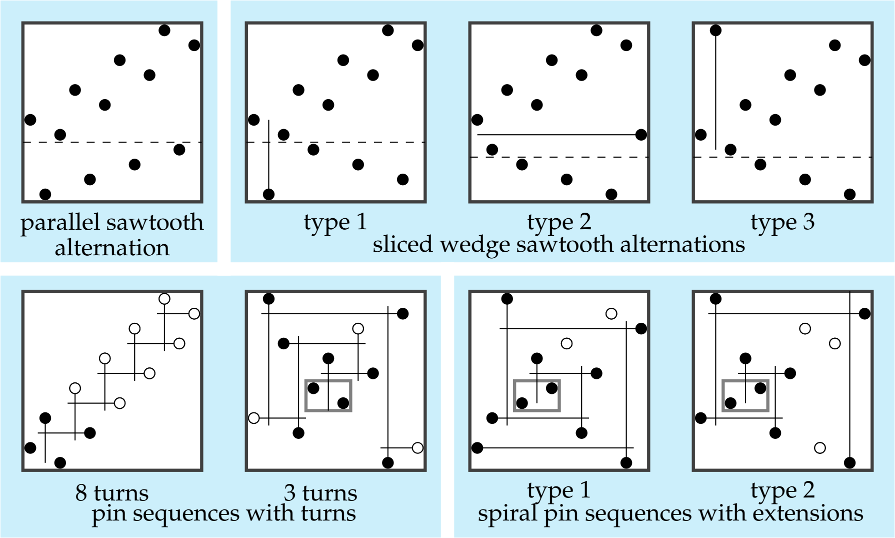
Theorem (I've already shown you this)

There exists a function $f(n)$ such that every simple permutation that contains a sum of $f(n)$ copies of 21 must contain a parallel or wedge sawtooth alternation of length $3n$ or an increasing oscillation of length n.

Proposition

Whenever a simple permutation contains a long wedge sawtooth alternation, then it contains a long split wedge sawtooth alternation, a proper pin sequence with many turns, or a spiral pin sequence with many extensions.

The basic simples



Closing remarks

- Q: Is this a decision procedure?

  A: Not quite, but it can probably be turned into one.

- ‘Geometrically griddable’ largely remains a remote goal (both for simple and generic permutations)

Takk!

Full paper: arXiv:1702.04269.